Let p and q be real numbers and let f be the function defined by:

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2, & \text{for } x \le 1 \\ qx + p, & \text{for } x > 1. \end{cases}$$

$1 + 2px - 2p + x^2 - 2x + 1$

$1 + \frac{2}{3}x - \frac{4}{3} + x^2 - 2x + 1$

(a) Find the value of q, in terms of p, for which f is continuous at $x = 1$.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$1 + 2p(1-1) + (1-1)^2 = q(1) + p$$

$$1 = q + p$$

$$\boxed{q = 1 - p}$$

(b) Find the values of p and q for which f is differentiable at $x = 1$.

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$$

$$\lim_{x \to 1^-} 2p + 2x - 2 = q$$

$$2p = q$$

$2p = 1 - p$

$3p = 1$

$\boxed{p = 1/3}$

$q = 2p$

$\boxed{q = 2/3}$

(c) If p and q have the values determined in part (b), is f'' a continuous function? Justify your answer.

$f(x)$ $\quad 1 + \frac{2}{3}x - \frac{4}{3} + x^2 - 2x + 1 \quad\quad \frac{2}{3}x + \frac{1}{3}$

$f'(x)$ $\quad \frac{2}{3} + 2x - 2 \quad\quad \frac{2}{3}$

$f''(x)$ $\quad 2 \quad \neq \quad 0$

Since $\lim_{x \to 1^-} f''(x) \neq \lim_{x \to 1^+} f''(x)$, $f''(x)$ is not continuous at $x=1$

Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

let $u = x^2$, then $f(u) = u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$u = 2 \quad u = 1$

$u = x^2 = 1 \quad\quad u = x^2 = 2$

$x = \pm 1 \quad\quad x = \pm\sqrt{2}$

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$.

$(x, f(x)) = (1, f(1)) = (1, 0) \quad f'(x) = 4x^3 - 6x \quad f'(1) = 4 - 6 = -2$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -2(x-1) \Rightarrow y = -2(x-1) = -2x + 2$$

(c) Find the x-coordinate of each point at which the line tangent to the graph of f is parallel to the line $y = -2x + 4$. since $m = -2$, find all x such that $f'(x) = -2$

$f'(x) = 4x^3 - 6x = -2$

$4x^3 - 6x + 2 = 0$

$2(2x^3 - 3x + 1) = 0$

$2x^3 - 3x + 1 = 0$

1	2	0	-3	1
		2	2	-1
	2	2	-1	$\boxed{0}$

$(x-1)(2x^2 + 2x - 1)$

$\boxed{x = 1}$

$D = b^2 - 4ac$

$= 4 - 4(2)(-1)$

$= 12$

$\sqrt{D} = \sqrt{12} = 2\sqrt{3}$

$x = \frac{-2 \pm 2\sqrt{3}}{4}$

$\boxed{x = \frac{1 \pm \sqrt{3}}{2}}$

Let f be the real-valued function defined by $f(x) = \sin^3 x + \sin^3 |x|$.

$$= \begin{cases} \sin^3 x + \sin^3 x, & x \geq 0 \\ \sin^3 x + \sin^3(-x), & x < 0 \end{cases}$$

remember: $\sin(-x) = -\sin(x)$

$$= \begin{cases} 2\sin^3 x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(a) Find $f'(x)$ for $x > 0$.

(b) Find $f'(x)$ for $x < 0$.

$$f'(x) = \begin{cases} 6\sin^2 x \cos x, & x \geq 0 \quad \text{a)} \\ 0, & x < 0 \quad \text{b)} \end{cases}$$

(c) Determine whether $f(x)$ is continuous at $x = 0$. Justify your answer.

continuous if,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

$$0 \stackrel{?}{=} 0 \quad \therefore \text{ yes, continuous}$$

(d) Determine whether the derivative of $f(x)$ exists at $x = 0$. Justify your answer.

derivative exists if,

$\lim_{x \to 0^-} f'(x) = \underline{0}$ is equal to $\lim_{x \to 0^+} f'(x) = \underline{0}$ $\therefore$ derivative exists!

Let f be the real-valued function defined by $f(x) = \sqrt{1 + 6x}$. $(1+6x)^{1/2}$

(a) Give the domain and range of f.

Domain: $1 + 6x \geq 0$, $6x \geq -1$, $x \geq -1/6$

Range: $[0, \infty)$

(b) Determine the slope of the line tangent to the graph of f at $x = 4$. find $f'(4)$

$$f'(x) = \frac{1}{2}(1+6x)^{-1/2}(6) = 3(1+6x)^{-1/2} = \frac{3}{\sqrt{1+6x}}, \quad f'(4) = \frac{3}{\sqrt{25}} = \frac{3}{5}$$

(c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.

$x = 4 \quad y = 5 \quad m = 3/5 \qquad y - 5 = \frac{3}{5}(x - 4) \Rightarrow y = \frac{3}{5}x - \frac{12}{5} + \frac{25}{5} = \frac{3}{5}x + \frac{13}{5}$

$\therefore$ y-intercept is $13/5$

(d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$. Since $m = 1$, find $f'(x) = 1$

$$f'(x) = \frac{3}{\sqrt{1+6x}} = \frac{1}{1}$$

$$(\sqrt{1+6x})^2 = (3)^2$$

$1 + 6x = 9$

$6x = 8$

$x = 8/6 = 4/3$

$$\frac{3}{\sqrt{1+6(\frac{4}{3})}}$$

$$\frac{3}{\sqrt{1+8}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$$

$\therefore f'(x) = 1$ when $x = \frac{4}{3}$

Let f be the function defined by $f(x) = \dfrac{2x-5}{x^2-4}$.

(a) Find the domain of f. all reals where $x \neq 2$ $x \neq -2$

(b) Write an equation for each vertical and each horizontal asymptote for the graph of f.

Vertical: $x=2$
$x=-2$

Horizontal: $\lim_{x\to\infty} \dfrac{2x-5}{x^2-4} = 0$ $\lim_{x\to-\infty} \dfrac{2x-5}{x^2-4} = 0$

so $y=0$ is the horizontal asymptote

(c) Find $f'(x)$.

$$f'(x) = \frac{(x^2-4)(2)-(2x-5)(2x)}{(x^2-4)^2} = \frac{2x^2-8-4x^2+10x}{(x^2-4)^2} = \frac{-2x^2+10x-8}{(x^2-4)^2}$$

$$= \frac{-2(x^2-5x+4)}{(x^2-4)^2}$$

$$= \frac{-2(x-4)(x-1)}{(x^2-4)^2}$$

$x=0$ $y=5/4$ $m=-1/2$

$y-5/4 = -\frac{1}{2}x$ or $y = -\frac{1}{2}x + \frac{5}{4}$

(d) Write an equation for the line tangent to the graph of f at the point $(0, f(0))$.

Let $f(x) = \sqrt{1-\sin x}$. defined when $1-\sin x \geq 0$
$\sin x \leq 1$ always true

(a) What is the domain of f? all reals

(b) Find $f'(x)$.

$$f'(x) = \frac{1}{2}(1-\sin x)^{-1/2}(-\cos x) = \frac{-\cos x}{2\sqrt{1-\sin x}}$$

(c) What is the domain of f'? Not defined when $1-\sin x = 0$
$\sin x = 1$
i.e. @ $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

(d) Write an equation for the line tangent to the graph of f at $x=0$.

$x=0$ $y=1$ $m=-1/2$

$y-1 = -\frac{1}{2}x$

$y = -\frac{1}{2}x + 1$

Let f be the function given by $f(x)=\sqrt{x^4-16x^2}$.

$x^4-16x^2 \geq 0$

$x^2(x^2-4) \geq 0$

$x^2(x+2)(x-2) \geq 0$

(a) Find the domain of f. $(-\infty,-2]\cup[2,\infty)$

(b) Describe the symmetry, if any, of the graph of f.

$f(x)$: $+$ on -3; $-$ on -1; $-$ on 1; $+$ on 3 (marks at -2, 0, 2)

Even if $f(-x)=f(x)$ (symmetric about y-axis)

Odd if $f(-x)=-f(x)$ (symmetric about origin)

$f(-x)=\sqrt{(-x)^4-16(-x)^2}=\sqrt{x^4-16x^2}$

$\therefore$ function is even and symmetric about the y-axis

(c) Find $f'(x)$.

$f'(x)=\frac{1}{2}(x^4-16x^2)^{-1/2}(4x^3-32x)$

$=\dfrac{4x(x^2-8)}{2\sqrt{x^4-16x^2}}=\boxed{\dfrac{2x(x^2-8)}{\sqrt{x^4-16x^2}}}$

$f'(5)=\dfrac{10(17)}{\sqrt{625-400}}=\dfrac{170}{15}=\dfrac{34}{3}$

$m_\perp=-\dfrac{3}{34}$

(d) Find the slope of the line normal to the graph of f at $x=5$.

A particle moves along the x-axis in such a way that its position at time t for $t\geq 0$ is given by $x=\frac{1}{3}t^3-3t^2+8t$.

$x(2)=\frac{1}{3}(2)^3-3(2)^2+8(2)=\frac{8}{3}-12+16=\frac{8}{3}+4=\frac{8}{3}+\frac{12}{3}=\frac{20}{3}$

(a) Show that at time $t=0$, the particle is moving to the right.

moving right when $v(t)>0$, so show $v(0)>0$.

$v(t)=t^2-6t+8$ and $v(0)=8$

Since $v(0)=8$, moving right at $t=0$

(b) Find all values of t for which the particle is moving to the left.

$v(t)=t^2-6t+8=(t-4)(t-2)=0$ when $t=4, t=2$

For $2<t<4$, particle moves left.

$v(t)$: $+$ on 1; $-$ on 3; $+$ on 5 (marks at 2, 4)

(c) What is the position of the particle at time $t=3$?

$x(3)=\frac{1}{3}(3)^3-3(3)^2+8(3)=-18+24=6$

t	$x(t)$
0	0
2	$20/3 = 6\frac{2}{3}$
3	6

(d) When $t=3$, what is the total distance the particle has traveled?

$6\frac{2}{3}+\frac{2}{3}=7\frac{1}{3}=\frac{22}{3}$

A particle starts at time $t = 0$ and moves on a number line so that its position at time t is given by $x(t) = \boxed{(t-2)^3}\boxed{(t-6)}$.

(a) When is the particle moving to the right? when $v(t) > 0$, i.e. $t > 5$

(b) When is the particle at rest? when $v(t) = 0$, i.e. when $t = 2$ or $t = 5$

(c) When does the particle change direction? when $v(t)$ changes sign, i.e @ $t = 5$

$$
\begin{aligned}
v(t) &= (t-2)^3(1) + (t-6)3(t-2)^2(1) \\
&= (t-2)^3 + 3(t-6)(t-2)^2 \\
&= (t-2)^2[t-2+3t-18] \\
&= (t-2)^2(4t-20) \\
&= 4(t-2)^2(t-5)
\end{aligned}
$$

v(t) – – +

1 2 3 5 6

(d) What is the farthest to the left of the origin that the particle moves?

Since $x(0) = 48$ and $x(5) = -27$ and $v(t) < 0$ for $0 < t < 5$ and $v(t) > 0$ for $t > 5$ then $x = -27$ is the farthest left of the origin that the particle moves

A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

(a) Find the velocity at time t. $v(t) = 2\pi - \sin(2\pi t)\cdot 2\pi = 2\pi(1 - \sin 2\pi t)$

(b) Find the acceleration at time t. $a(t) = -4\pi^2 \cos(2\pi t)$

(c) What are all values of t, $0 \le t \le 3$, for which the particle is at rest?

when $v(t) = 0$, or whenever $1 - \sin 2\pi t = 0$ or $\sin 2\pi t = 1$

$\sin x = 1$ when $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \ldots$

$2\pi t = \frac{\pi}{2}$, $\underline{t = 1/4}$ $2\pi t = 5\pi/2$, $\underline{t = 5/4}$ $2\pi t = \frac{9\pi}{2}$, $\underline{t = 9/4}$ $\cancel{2\pi t = \frac{13\pi}{2}}$, $\cancel{t = 13/4}$

(d) What is the maximum velocity?

Find $a(t) = 0$ $a(t) = 0$ when $\cos(2\pi t) = 0$ $\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$2\pi t = \pi/2$ $2\pi t = \frac{3\pi}{2}$ $2\pi t = \frac{5\pi}{2}$

$t = 1/4$ $t = 3/4$ $t = 5/4$ $t = 7/4$

$t = 9/4$ $t = 11/4$

t	$1/4$	$3/4$	$5/4$	$7/4$	$9/4$	$11/4$
$v(t)$	0	4π	0	4π	0	4π

Max velocity is 4π.