AP Calculus AB Mid-Term Review Problem Set

1.

$y' = x^2 + 10x$
$y'' = 2x + 10 = 0$ when $x = -5$

What is the $x$-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

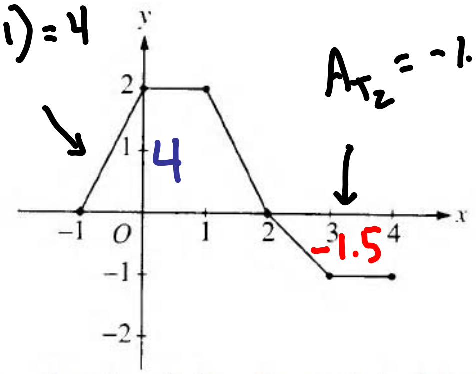
$y''$ changes sign at $x = -5$

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) $-5$ (E) $-10$

2.

$A_{T_1} = \frac{2}{2}(3+1) = 4$

$A_{T_2} = -1.5$




The graph of a piecewise-linear function $f$, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3.

$\int_1^2 x^{-2}\,dx = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$

$\int_1^2 \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

4.

If $f$ is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

MVT (A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some $c$ such that $a < c < b$.

X (B) $f'(c) = 0$ for some $c$ such that $a < c < b$. ← Rolle's Theorem, but this requires that $f(b) = f(a)$ as well

EVT (C) $f$ has a minimum value on $a \le x \le b$.

EVT (D) $f$ has a maximum value on $a \le x \le b$.

FTC (E) $\int_a^b f(x)dx$ exists.

5.

$\int_0^x \sin t\, dt = \left[-\cos t\right]_0^x = -\cos x - (-\cos 0) = -\cos x + 1$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

6.

plug in 2 and solve for y

If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

$2^2 + 2y = 10$
$2y = 6$
$y = 3$

(A) $-\dfrac{7}{2}$ (B) $-2$ (C) $\dfrac{2}{7}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

$2x + xy' + y = 0$
$2(2) + 2y' + 3 = 0$
$2y' = -7$
$y' = -\dfrac{7}{2}$

7.

Let $f$ and $g$ be differentiable functions with the following properties:

(i) $g(x) > 0$ for all $x$

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) $e^x$ (D) $0$ (E) $1$

$h(x) = f(x)g(x)$

product rule

$h'(x) = f(x)g'(x) + g(x)f'(x) \overset{\text{but}}{=} f(x)g'(x)$

therefore $g(x)f'(x) = 0$, but $g(x) > 0$

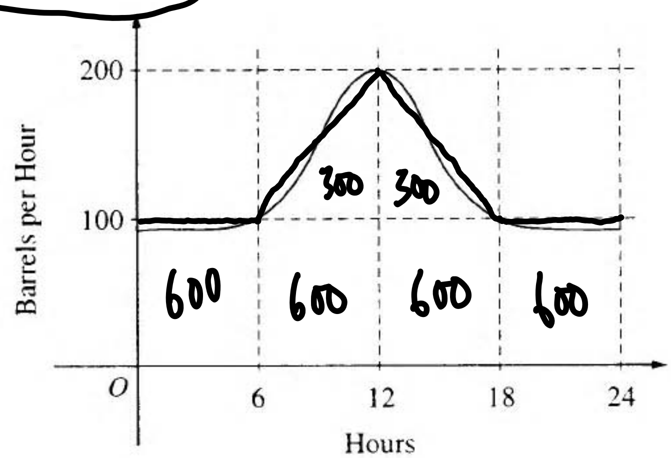
so $f'(x) \overset{\text{must be}}{=} 0$

then $f(x)$ should be a horizontal line

since $f(0) = 1$, $f(x) = 1$

and we used all the given info

8.




The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

9.

What is the instantaneous rate of change at $x = 2$ of the function $f$ given by $f(x) = \dfrac{x^2 - 2}{x - 1}$ ?

quotient rule

(A) $-2$ $x=2$ (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

don't spend time simplifying after the quotient rule

$$f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2} = \frac{(1)(4) - (2)(1)}{(1)^2} = \frac{2}{1} = 2$$

10.

If $f$ is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

If $f$ is linear, $f'(x)$ is a constant function and $f''(x)$ is 0

(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b - a$ (E) $\dfrac{b^2 - a^2}{2}$

the area between 0 and the x-axis is 0

$\lim_{x \to 2^-} f(x) = \ln 2$

$\lim_{x \to 2^+} f(x) = 4 \ln 2$

not equal

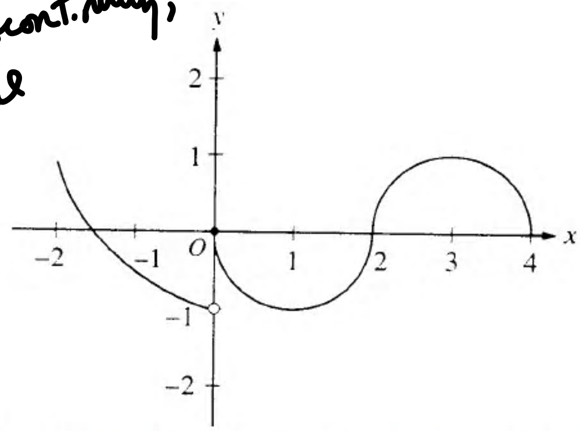
11.

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

12.

A function is not differentiable whenever there is a discontinuity, a sharp turn, or a vertical tangent




The graph of the function $f$ shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of $x$, $-2 < x < 4$, is $f$ not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

13.

A particle moves along the $x$-axis so that its position at time $t$ is given by $x(t) = t^2 - 6t + 5$. For what value of $t$ is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$v(t) = 2t - 6 = 0$ when $t = 3$

14.

If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) −3 (B) −2 (C) 2 (D) 3 (E) 18

$F'(x) = \frac{d}{dx}\left[\int_0^x \sqrt{t^3+1}\, dt\right] = \sqrt{x^3+1}$

$F'(2) = \sqrt{9} = 3$

15. triple chain rule !!

If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x})+e^{-x}$

(C) $\cos(e^{-x})-e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

$$\frac{d}{dx}\left[\sin\left(e^{-x}\right)\right] = \cos\left(e^{-x}\right)\frac{d}{dx}\left[e^{-x}\right]$$

$$= \cos\left(e^{-x}\right)\left(e^{-x}\right)\frac{d}{dx}\left[-x\right]$$

$$= \cos\left(e^{-x}\right)\left(e^{-x}\right)(-1)$$

$$= -e^{-x}\cos\left(e^{-x}\right)$$

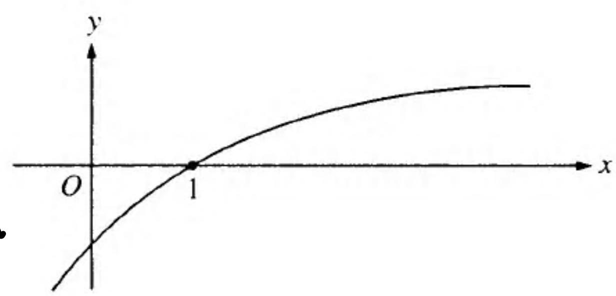
16.

$f(1)=0$

$f'(1)>0$ increasing

$f''(1)<0$ concave down

so $\underset{-}{f''(1)} < \underset{0}{f(1)} < \underset{+}{f'(1)}$




The graph of a twice-differentiable function $f$ is shown in the figure above. Which of the following is true?

(A) $f(1)<f'(1)<f''(1)$

(B) $f(1)<f''(1)<f'(1)$

(C) $f'(1)<f(1)<f''(1)$

(D) $f''(1)<f(1)<f'(1)$

(E) $f''(1)<f'(1)<f(1)$

17.

$y' = 1 - \sin x$ @ $x = 0$, $y' = 1$

An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x+1$ (B) $y = x+1$ (C) $y = x$ (D) $y = x-1$ (E) $y = 0$

$m = y'(0) = 1$ $x = 0$ $y = 1$

$y - 1 = 1(x - 0)$

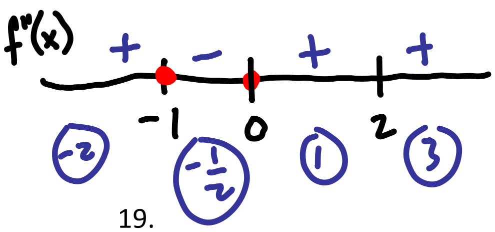
$\boxed{y = x+1}$

18.

$x=0$ $x=-1$ $x=2$

If $f''(x) = x(x+1)(x-2)^2$, then the graph of $f$ has inflection points when $x =$

(A) $-1$ only (B) 2 only (C) $-1$ and 0 only (D) $-1$ and 2 only (E) $-1$, 0, and 2 only




19.

What are all values of $k$ for which $\int_{-3}^{k} x^2\,dx = 0$?

(A) $-3$ (B) 0 (C) 3 (D) $-3$ and 3 (E) $-3$, 0, and 3

$$\int_{-3}^{k} x^2\,dx = \left[\frac{x^3}{3}\right]_{-3}^{k} = \frac{1}{3}\left[k^3 - (-27)\right] = \frac{1}{3}(k^3+27) = 0$$

when $k^3 = 27$

$k = -3$

but this makes sense, because

$\int_{-3}^{-3} x^2\,dx = 0$ right??

and because $x^2 \geq 0$ for all $x$, $\int_{-3}^{k} x^2\,dx$ won't be 0 for any other value of $k$.

20.

$f'(x) = 4x^3 + 2x$

The function $f$ is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is $f$ increasing?

$f'(x) > 0$ when $4x^3 + 2x > 0$

$2x(2x^2 + 1) > 0$

$x = 0$ always $> 0$

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

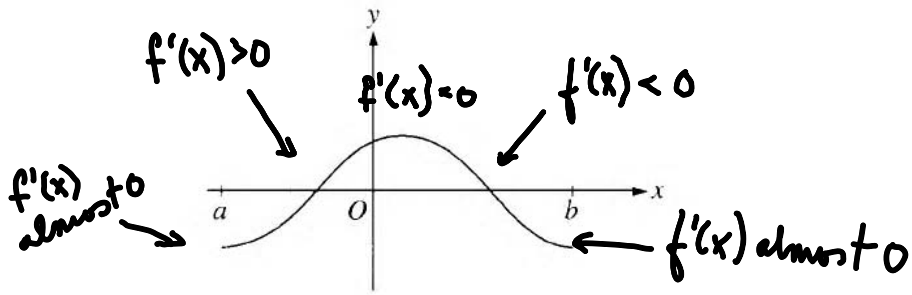
(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x)$ — +

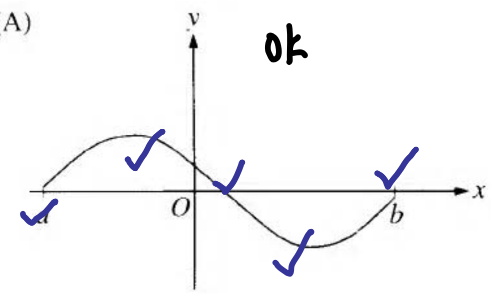
-1 0 +2

21.

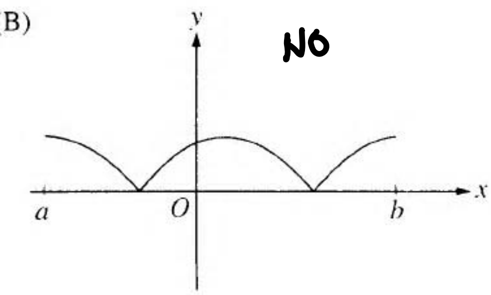



The graph of $f$ is shown in the figure above. Which of the following could be the graph of the derivative of $f$?

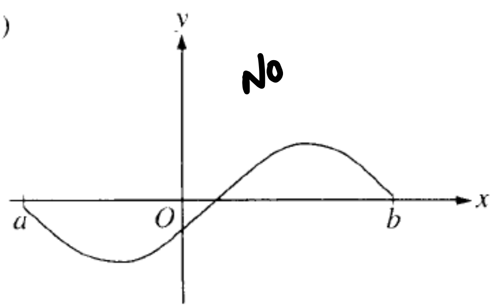
(A)



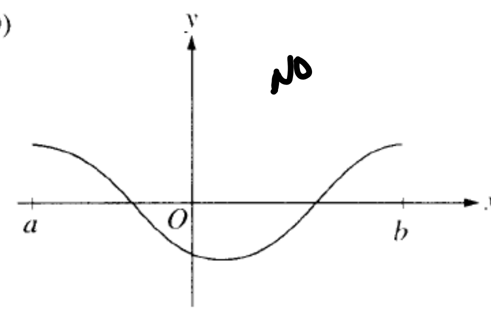

(B)



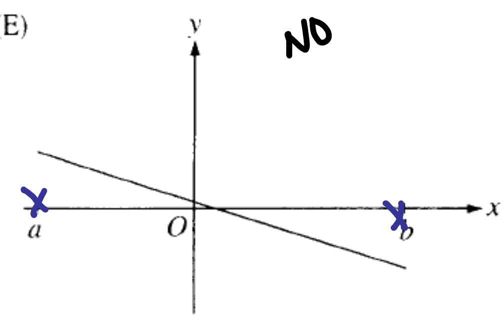

(C)




(D)




(E)



22. closed interval endpoints and critical value of a(t)

abs. max

The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

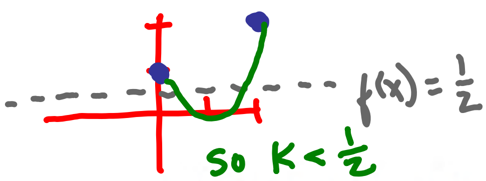
(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6 = 0$ when $t = 1$

| $t$ | $a(t)$ |
|---|---|
| 0 | 12 |
| 1 | 9 |
| 3 | 21 ← abs max |

23.




so $K < \frac{1}{2}$

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |

The function $f$ is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

only possible correct answer

24.

If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

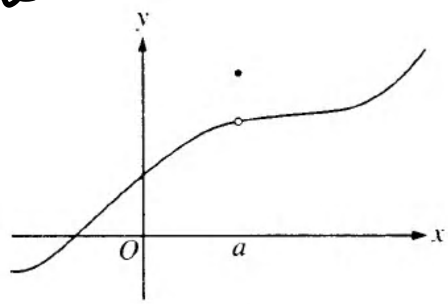
$f'(x) = 2\sec^2(2x) = \dfrac{2}{\cos\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{3}\right)} = \dfrac{2}{\frac{1}{2} \cdot \frac{1}{2}} = \dfrac{2}{\frac{1}{4}} = 8$

Calculator

25.

Even though $f(a)$ exists, it is not continuous there

$\lim_{x \to a} f(x) \neq f(a)$




The graph of a function $f$ is shown above. Which of the following statements about $f$ is false?

(A) $f$ is continuous at $x = a$.

(B) $f$ has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of $f$.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

26.

Let $f$ be the function given by $f(x) = 3e^{2x}$ and let $g$ be the function given by $g(x) = 6x^3$. At what value of $x$ do the graphs of $f$ and $g$ have parallel tangent lines?

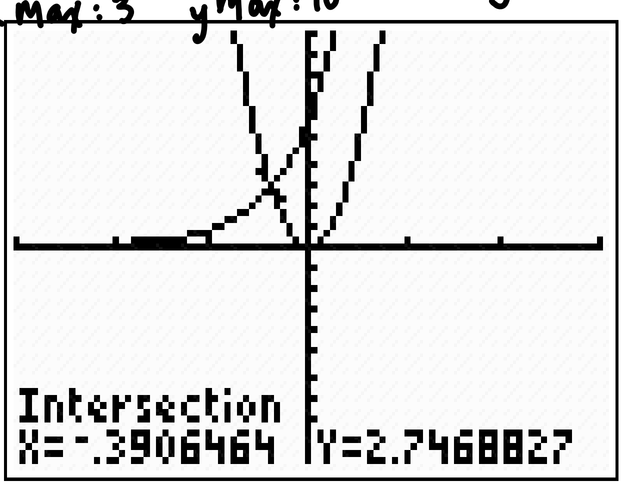
(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258

f and g have parallel tangent lines when $f'(x) = g'(x)$

$f'(x) = 3e^{2x}(2) = 6e^{2x}$

$g'(x) = 18x^2$

xMin: -3 yMin: -10
xMax: 3 yMax: 10



27.

The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference $C$, what is the rate of change of the area of the circle, in square centimeters per second?

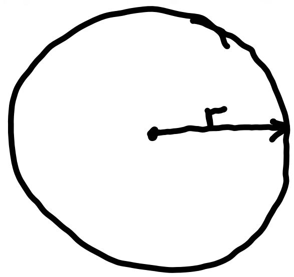
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$



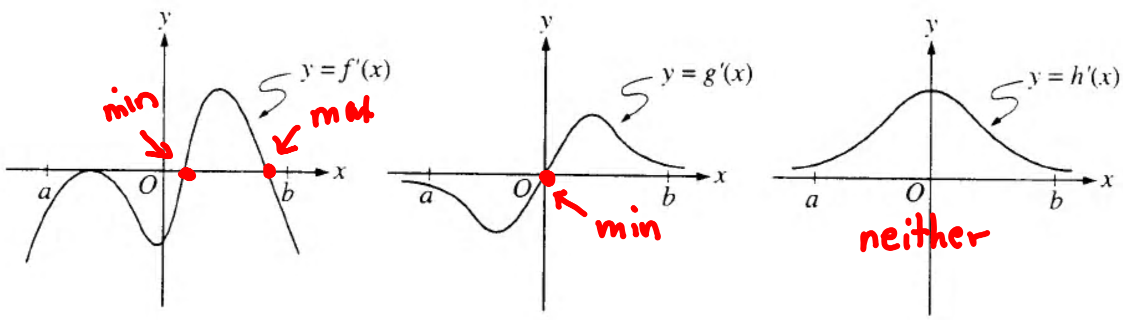

$\frac{dr}{dt} = -0.1$ cm/sec

$A = \pi r^2$

$A' = 2\pi r \frac{dr}{dt}$ $\quad C = 2\pi r$

$A' = C\frac{dr}{dt} = -0.1C$

28.




The graphs of the derivatives of the functions $f$, $g$, and $h$ are shown above. Which of the functions $f$, $g$, or $h$ have a relative maximum on the open interval $a < x < b$?

(A) $f$ only
(B) $g$ only
(C) $h$ only
(D) $f$ and $g$ only
(E) $f$, $g$, and $h$

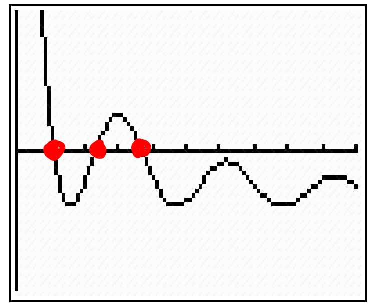
29.

graph the derivative and look for zeros.

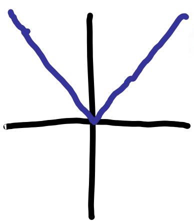
The first derivative of the function $f$ is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does $f$ have on the open interval $(0,10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

xMin: 0
xMax: 10
yMin: -.5
yMax: .5



30.



Let $f$ be the function given by $f(x) = |x|$. Which of the following statements about $f$ are true?

I. $f$ is continuous at $x = 0$. ✓
II. $f$ is differentiable at $x = 0$. ✗
III. $f$ has an absolute minimum at $x = 0$. ✓

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

BC only 31.

If $f$ is a continuous function and if $F'(x) = f(x)$ for all real numbers $x$, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$
$u(3) = 6$
$u(1) = 2$

$\frac{1}{2}\int_2^6 f(u)\,du$

$\frac{1}{2}[F(6) - F(2)]$

32.

If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

$$\frac{\cancel{(x+a)(x-a)}}{(x^2+a^2)\cancel{(x^2-a^2)}} = \lim_{x \to a} \frac{1}{x^2+a^2} = \frac{1}{a^2+a^2} = \frac{1}{2a^2}$$

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

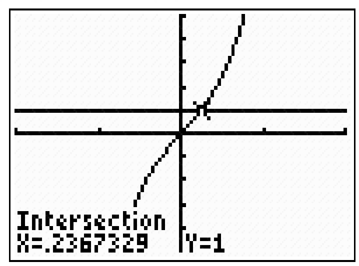
33.

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

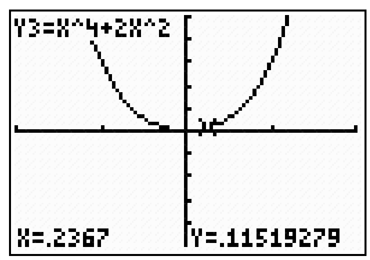
(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

$f'(x) = 4x^3 + 4x = 1$
when $x = .2367$
$y = .1152$

$y - .1152 = 1(x - .2367)$
$y = x - .2367 + .1152$
$y = x - .122$




1) graph $f'(x)$ and $y = 1$ to find $x$-coordinate of intersection




2. graph $f(x)$ and find the $y$-value

Now:
$m = 1$
$x = .2367$
$y = .1152$

34.

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

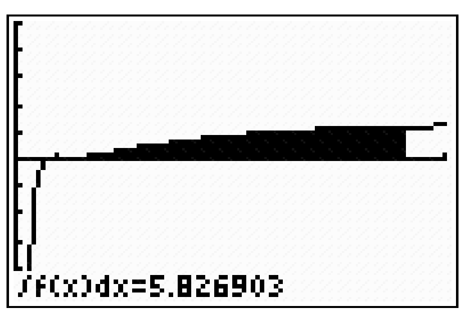
(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

$$\int_1^9 \frac{(\ln x)^3}{x}\,dx = F(9) - \cancel{F(1)}^{\,0}$$

use calculator to find this

5.827

given

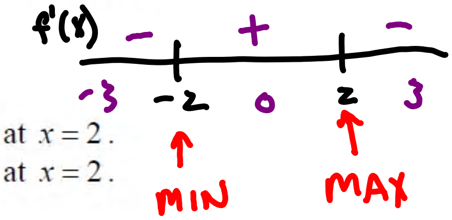


35.

always negative

If $g$ is a differentiable function such that $g(x) < 0$ for all real numbers $x$ and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

$(x+2)(x-2)$




(A) $f$ has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) $f$ has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) $f$ has relative minima at $x = -2$ and at $x = 2$.
(D) $f$ has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if $f$ has any relative extrema.

36.

If the base $b$ of a triangle is increasing at a rate of 3 inches per minute while its height $h$ is decreasing at a rate of 3 inches per minute, which of the following must be true about the area $A$ of the triangle?

(A) $A$ is always increasing.
(B) $A$ is always decreasing.
(C) $A$ is decreasing only when $b < h$.
(D) $A$ is decreasing only when $b > h$.
(E) $A$ remains constant.

$\frac{db}{dt} = 3$ in/min $\qquad \frac{dh}{dt} = -3$ in/min

$A = \frac{1}{2}bh$ ← product rule!!

$$\frac{dA}{dt} = \frac{1}{2}\left[b\frac{dh}{dt} + h\frac{db}{dt}\right]$$

$$= \frac{1}{2}\left[-3b + 3h\right]$$

$$= \frac{3}{2}\left[h - b\right]$$

positive when $h > b$
negative when $b > h$

A decreases when $b > h$

37.

smooth curve

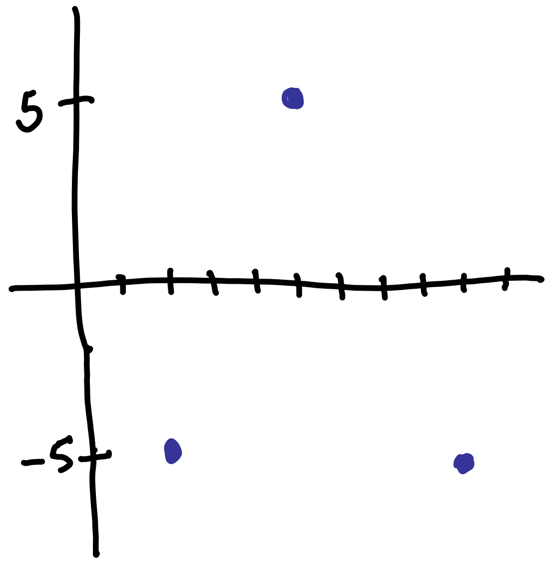
Let $f$ be a function that is differentiable on the open interval $(1,10)$. If $f(2)=-5$, $f(5)=5$, and $f(9)=-5$, which of the following must be true?

IVT ✓ I. $f$ has at least 2 zeros.

Rolle's Theorem II. ✓ The graph of $f$ has at least one horizontal tangent.

IVT III. For some $c$, $2<c<5$, $f(c)=3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III




38.

If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y=\cos x$ from $x=k$ to $x=\frac{\pi}{2}$ is 0.1, then $k=$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

$$\int_k^{\pi/2} \cos x \, dx = \left[\sin x\right]_k^{\pi/2} = \sin\frac{\pi}{2} - \sin k = 0.1$$

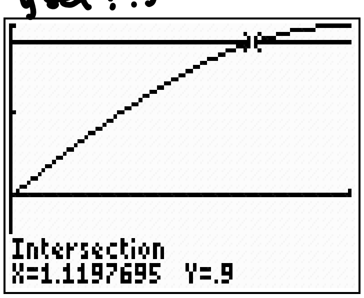
$$1 - \sin k = 0.1$$

$$\sin k = .9$$

Xmin: 0
X max: $\pi/2$
y min: -.5
y max: 1
y scl: .5

graph sin x and y=.9
look for pt. of intersection
on $0 \le x < \frac{\pi}{2}$

or use $\sin^{-1}(.9)$




```
sin⁻¹(.9)
     1.119769515
```