

AP Calculus BC Mid-Term Review Problem Set

1.

What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

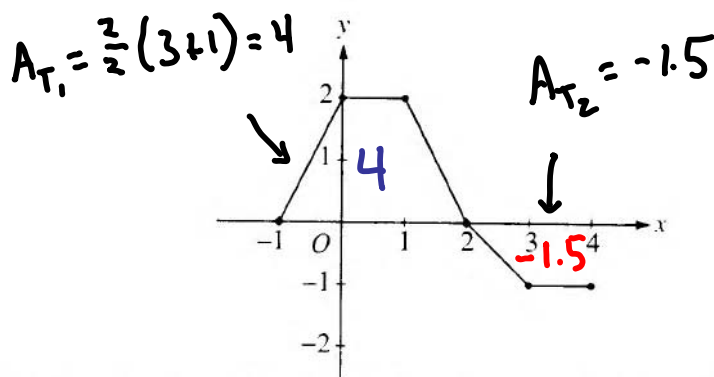
$$y' = x^2 + 10x$$

$$y'' = 2x + 10 = 0 \text{ when}$$

y'' changes sign at $x = -5$

- (A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

2.



The graph of a piecewise-linear function f , for $-1 \leq x \leq 4$, is shown above. What is the value of

$$\int_{-1}^4 f(x) dx?$$

- (A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3.

$$\int_1^2 x^{-2} dx = \left[-\frac{1}{x} \right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

$$\int_1^2 \frac{1}{x^2} dx =$$

- (A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

4.

If f is continuous for $a \leq x \leq b$ and differentiable for $a < x < b$, which of the following could be false?

MVT (A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

X (B) $f'(c) = 0$ for some c such that $a < c < b$. ← Rolle's Theorem, but this requires that $f(b) = f(a)$ as well

EVT (C) f has a minimum value on $a \leq x \leq b$.

EVT (D) f has a maximum value on $a \leq x \leq b$.

FTC (E) $\int_a^b f(x) dx$ exists.

5.

$$\int_0^x \sin t dt = [-\cos t]_0^x = -\cos x - (-\cos 0) = -\cos x + 1$$

(A) $\sin x$

(B) $-\cos x$

(C) $\cos x$

(D) $\cos x - 1$

(E) $1 - \cos x$

6.

plug in 2 and solve for y

If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$$\begin{aligned} 2^2 + 2y &= 10 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

(A) $-\frac{7}{2}$

(B) -2

(C) $\frac{2}{7}$

(D) $\frac{3}{2}$

(E) $\frac{7}{2}$

$$2x + xy' + y = 0$$

$$2(2) + 2y' + 3 = 0$$

$$2y' = -7$$

$$y' = -\frac{7}{2}$$

BC only

7. Simplify first

$$\int_1^e \left(\frac{x^2-1}{x} \right) dx = \int_1^e \left(x - \frac{1}{x} \right) dx = \left[\frac{x^2}{2} - \ln x \right]_1^e = \left[\left(\frac{e^2}{2} - 1 \right) - \left(\frac{1}{2} - 0 \right) \right] = \frac{e^2}{2} - \frac{3}{2}$$

- (A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

8.

Let f and g be differentiable functions with the following properties:

- (i) $\frac{g(x) > 0}{f(0) = 1}$ for all x
(ii) $\frac{g(x) > 0}{f(0) = 1}$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

- (A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

Product rule

$$h(x) = f(x)g(x)$$

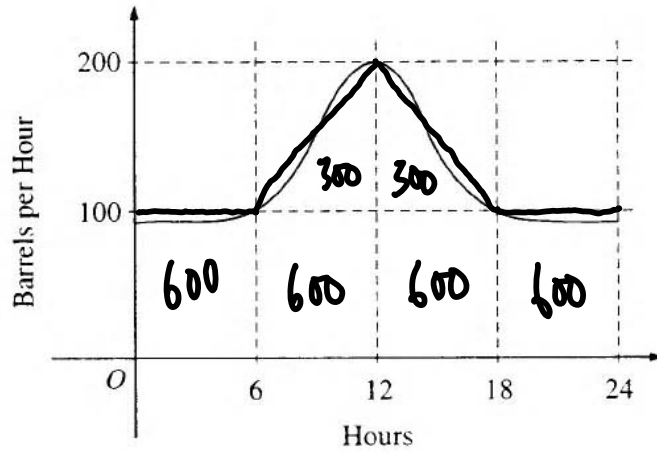
$$h'(x) = f(x)g'(x) + g(x)f'(x) \quad \text{but} \quad f(x)g'(x)$$

therefore $g(x)f'(x) = 0$, but $g(x) > 0$
so $f'(x) = 0$ must be

then $f(x)$ should be a horizontal line
since $f(0) = 1$, $f(x) = 1$

and we used all the given info

9.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

- (A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

10.

What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

quotient rule

- (A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

don't spend time simplifying after the quotient rule

$$f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2} = \frac{(1)(4) - (2)(1)}{(1)^2} = \frac{2}{1} = 2$$

11.

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) dx =$
 If f is linear, $f'(x)$ is a constant function and $f''(x)$ is 0

- (A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

the area between 0 and the x-axis is 0

12.

$$\lim_{x \rightarrow 2^-} f(x) = \ln 2 \quad \swarrow \text{not equal}$$

$$\lim_{x \rightarrow 2^+} f(x) = 4 \ln 2$$

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then $\lim_{x \rightarrow 2} f(x)$ is

(A) $\ln 2$

(B) $\ln 8$

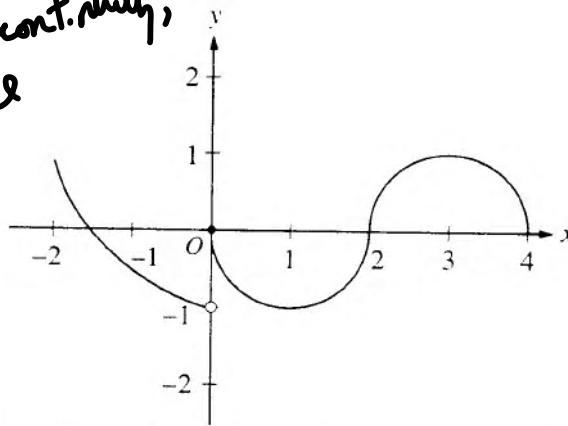
(C) $\ln 16$

(D) 4

(E) nonexistent

13.

A function is not differentiable whenever there is a discontinuity, a sharp turn, or a vertical tangent



The graph of the function f shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x , $-2 < x < 4$, is f not differentiable?

- (A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14.

A particle moves along the x -axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

$$v(t) = 2t - 6 = 0 \text{ when } t = 3$$

15.

If $F(x) = \int_0^x \sqrt{t^3+1} dt$, then $F'(2) =$

(A) -3

(B) -2

(C) 2

(D) 3

(E) 18

$$F'(x) = \frac{d}{dx} \left[\int_0^x \sqrt{t^3+1} dt \right] = \sqrt{x^3+1}$$

$$F'(2) = \sqrt{9} = 3$$

16.

triple chain rule !!

If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x} \cos(e^{-x})$

(E) $-e^{-x} \cos(e^{-x})$

$$\frac{d}{dx} [\sin(e^{-x})] = \cos(e^{-x}) \frac{d}{dx} [e^{-x}]$$

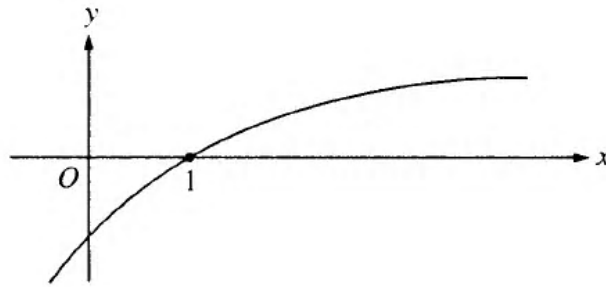
$$= \cos(e^{-x}) (e^{-x}) \frac{d}{dx} [-x]$$

$$= \cos(e^{-x}) (e^{-x}) (-1)$$

$$= -e^{-x} \cos(e^{-x})$$

17.

$f(1) = 0$
 $f'(1) > 0$ increasing
 $f''(1) < 0$ concave down
 so $f''(1) < f(1) < f'(1)$



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

- (A) $f(1) < f'(1) < f''(1)$
- (B) $f(1) < f''(1) < f'(1)$
- (C) $f'(1) < f(1) < f''(1)$
- (D) $f''(1) < f(1) < f'(1)$
- (E) $f''(1) < f'(1) < f(1)$

18.

$y' = 1 - \sin x$ @ $x=0$, $y' = 1$

An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

- (A) $y = 2x + 1$
- (B) $y = x + 1$
- (C) $y = x$
- (D) $y = x - 1$
- (E) $y = 0$

$m = y'(0) = 1$ $x = 0$ $y = 1$

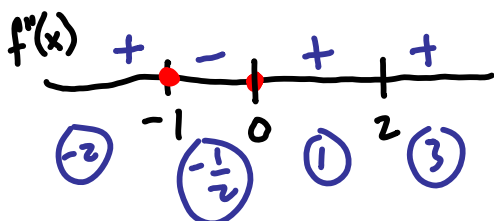
$y - 1 = 1(x - 0)$
 $y = x + 1$

19.

$x = 0$ $x = -1$ $x = 2$

If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

- (A) -1 only
- (B) 2 only
- (C) -1 and 0 only
- (D) -1 and 2 only
- (E) -1 , 0 , and 2 only



20.

What are all values of k for which $\int_{-3}^k x^2 dx = 0$?

- (A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) $-3, 0,$ and 3

$$\int_{-3}^k x^2 dx = \left[\frac{x^3}{3} \right]_{-3}^k = \frac{1}{3} [k^3 - (-27)] = \frac{1}{3} (k^3 + 27) = 0$$

when $k^3 = -27$
 $k = -3$

but this makes sense, because

$$\int_{-3}^{-3} x^2 dx = 0 \text{ right??}$$

and because $x^2 \geq 0$ for all x , $\int_{-3}^k x^2 dx$ won't be 0 for any other value of k .

21.

$$f'(x) = 4x^3 + 2x$$

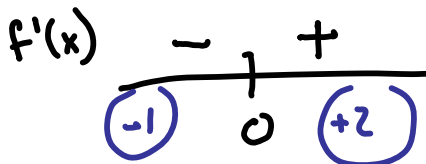
The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

- (A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
 (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
 (C) $(0, \infty)$
 (D) $(-\infty, 0)$
 (E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

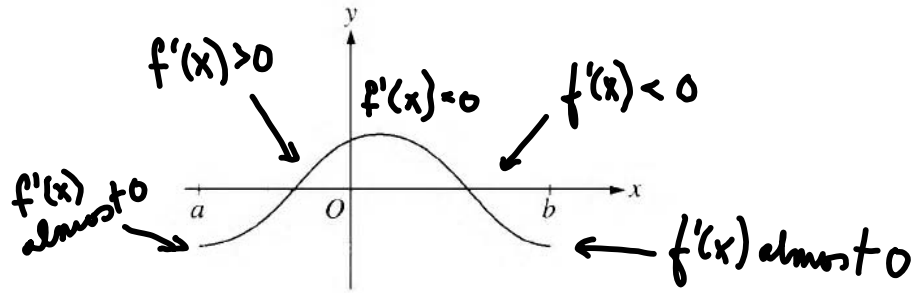
$$f'(x) > 0 \text{ when } 4x^3 + 2x > 0$$

$$2x(2x^2 + 1) > 0$$

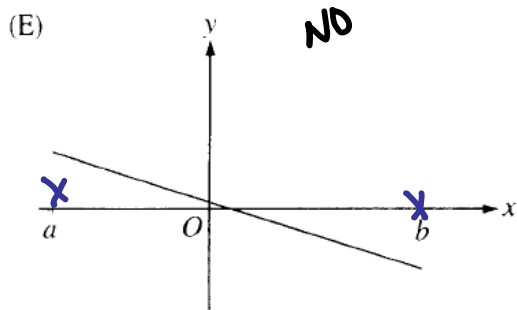
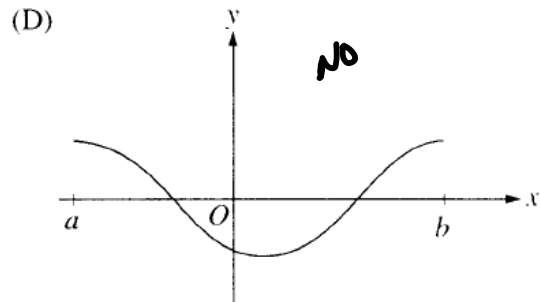
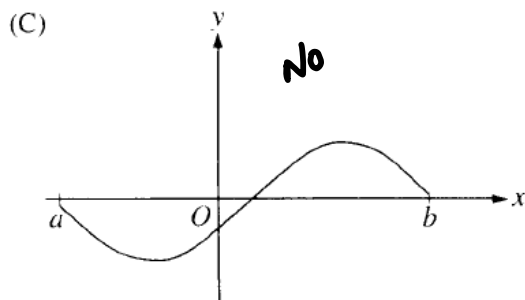
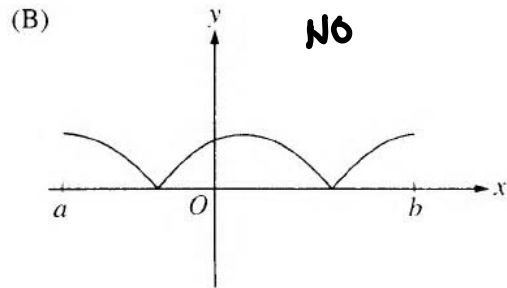
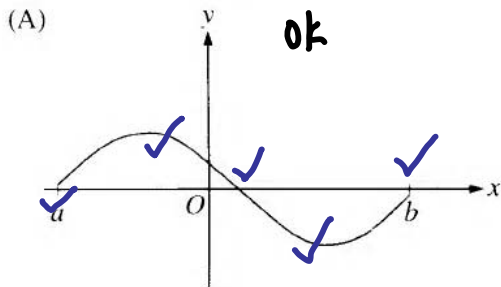
\uparrow \uparrow
 $x=0$ always > 0



22.



The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f ?



23. closed interval endpoints and critical value of $a(t)$

Abs. max

The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

- (A) 9 (B) 12 (C) 14 (D) 21 (E) 40

$$a(t) = 3t^2 - 6t + 12$$

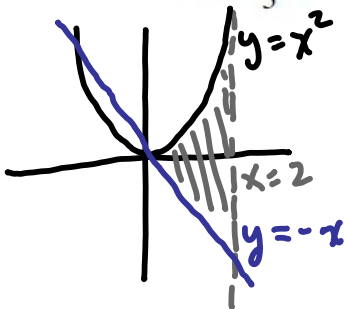
$$a'(t) = 6t - 6 = 0 \text{ when } t = 1$$

t	$a(t)$
0	12
1	9
3	21 ← abs max

24.

What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

- (A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$



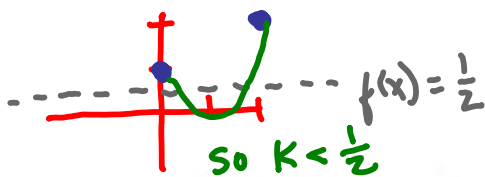
$$\int_0^2 x^2 - (-x) dx = \int_0^2 x^2 + x dx = \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_0^2$$

$$= \left(\frac{8}{3} + 2 \right) - 0$$

$$= \frac{14}{3}$$

Here we aren't looking for the area enclosed by the two graphs. Be careful!

25.



x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

only possible correct answer

26.

What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

BC
only

(A) $\frac{26}{9}$

(B) $\frac{52}{9}$

(C) $\frac{26}{3}$

(D) $\frac{52}{3}$

(E) 24

$$\frac{1}{2} \int_0^2 x^2 \sqrt{x^3+1} dx$$

$$\frac{1}{6} \int_1^9 u^{1/2} du$$

$$u = x^3 + 1$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$u(2) = 9$$

$$u(0) = 1$$

$$\frac{1}{6} \int_1^9 u^{1/2} du = \frac{1}{6} \left[\frac{2}{3} u^{3/2} \right]_1^9 = \frac{1}{9} \left[u^{3/2} \right]_1^9$$

$$= \frac{1}{9} [27 - 1] = \frac{26}{9}$$

27.

If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) 4

(D) $4\sqrt{3}$

(E) 8

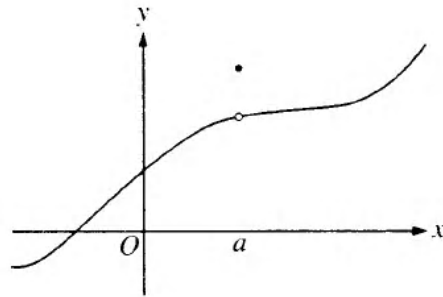
$$f'(x) = 2 \sec^2(2x) = \frac{2}{\cos\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{3}\right)} = \frac{2}{\frac{1}{2} \cdot \frac{1}{2}} = \frac{2}{\frac{1}{4}} = 8$$

Calculator

28.

Even though $f(a)$ exists,
it is not continuous there

$$\lim_{x \rightarrow a} f(x) \neq f(a)$$



The graph of a function f is shown above. Which of the following statements about f is false?

- (A) f is continuous at $x = a$.
- (B) f has a relative maximum at $x = a$.
- (C) $x = a$ is in the domain of f .
- (D) $\lim_{x \rightarrow a^+} f(x)$ is equal to $\lim_{x \rightarrow a^-} f(x)$.
- (E) $\lim_{x \rightarrow a} f(x)$ exists.

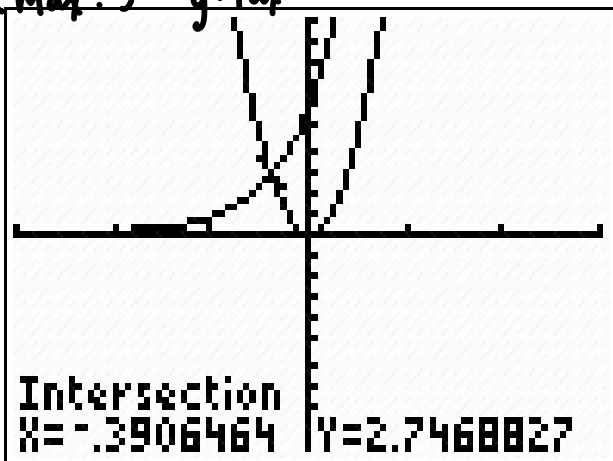
29.

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

- (A) -0.701
- (B) -0.567
- (C) -0.391
- (D) -0.302
- (E) -0.258

f and g have parallel tangent lines when $f'(x) = g'(x)$
 $f'(x) = 3e^{2x} (2) = 6e^{2x}$
 $g'(x) = 18x^2$

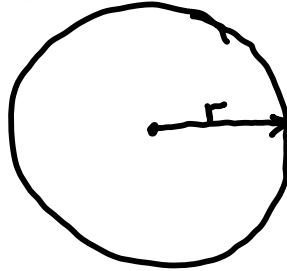
x Min: -3
x Max: 3
y Min: -10
y Max: 10



30.

The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C , what is the rate of change of the area of the circle, in square centimeters per second?

- (A) $-(0.2)\pi C$
- (B) $-(0.1)C$
- (C) $-\frac{(0.1)C}{2\pi}$
- (D) $(0.1)^2 C$
- (E) $(0.1)^2 \pi C$



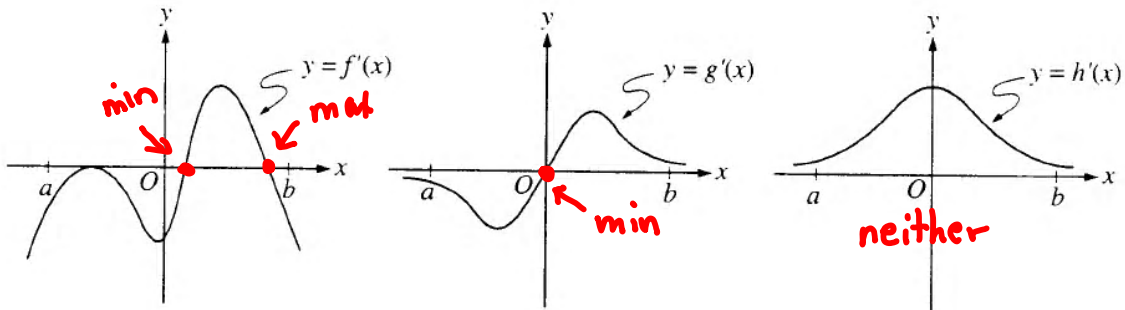
$$\frac{dr}{dt} = -0.1 \text{ cm/sec}$$

$$A = \pi r^2$$

$$A' = 2\pi r \frac{dr}{dt} \quad C = 2\pi r$$

$$A' = C \frac{dr}{dt} = -0.1 C$$

31.



The graphs of the derivatives of the functions f , g , and h are shown above. Which of the functions f , g , or h have a relative maximum on the open interval $a < x < b$?

- (A) f only
- (B) g only
- (C) h only
- (D) f and g only
- (E) f , g , and h

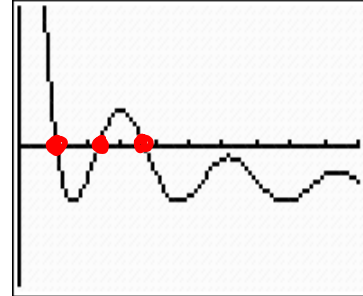
32.

graph the derivative and look for zeros.

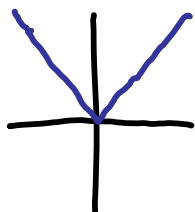
The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

- (A) One
- (B) Three
- (C) Four
- (D) Five
- (E) Seven

x Min: 0
 x Max: 10
 y Min: -.5
 y Max: .5



33.



Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

- I. f is continuous at $x = 0$. ✓
- II. f is differentiable at $x = 0$. ✗
- III. f has an absolute minimum at $x = 0$. ✓

- (A) I only
- (B) II only
- (C) III only
- (D) I and III only
- (E) II and III only

34.

If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x , then $\int_1^3 f(2x) dx =$

- (A) $2F(3) - 2F(1)$
- (B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
- (C) $2F(6) - 2F(2)$
- (D) $F(6) - F(2)$
- (E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

$u = 2x$
 $du = 2dx$
 $\frac{1}{2} du = dx$
 $u(3) = 6$
 $u(1) = 2$

$$\frac{1}{2} \int_2^6 f(u) du$$

$$\frac{1}{2} [F(6) - F(2)]$$

35.

If $a \neq 0$, then $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$ is

$$\frac{\cancel{(x+a)}(x-a)}{(x^2+a^2)\cancel{(x^2-a^2)}} = \lim_{x \rightarrow a} \frac{1}{x^2+a^2} = \frac{1}{a^2+a^2} = \frac{1}{2a^2}$$

- (A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

36.

x	2	5	7	8
$f(x)$	10	30	40	20

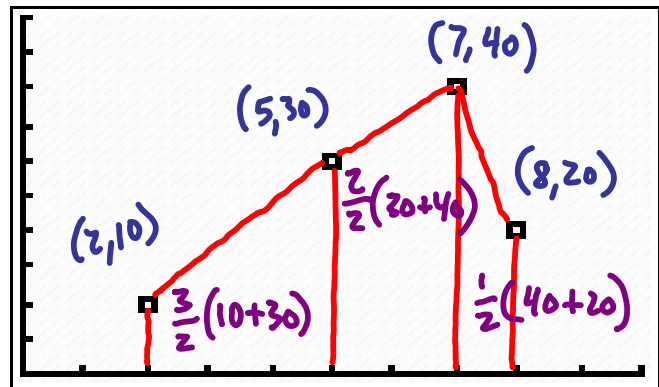
The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table above. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of

$$\int_2^8 f(x) dx?$$

- (A) 110 (B) 130 (C) 160 (D) 190 (E) 210

$$\int_2^8 f(x) dx \approx 60 + 70 + 30 = 160$$

x Min: 0
 x Max: 10
 y Min: 0
 y Max: 50
 y scl: 5



BC Only 37.

$$x + 2y = 8$$

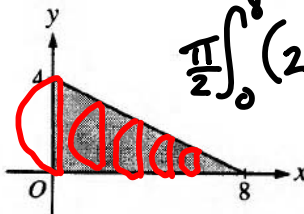
$$2y = -x + 8$$

$$y = -\frac{1}{2}x + 4$$

DIAMETER

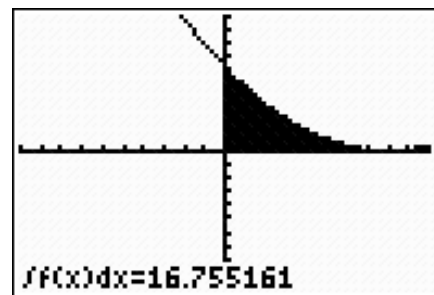
$$r = \frac{1}{2}(-\frac{1}{2}x + 4)$$

$$= -\frac{1}{4}x + 2$$



$$A = \frac{\pi r^2}{2}$$

$$\frac{\pi}{2} \int_0^8 (2 - \frac{1}{4}x)^2 dx$$



The base of a solid is a region in the first quadrant bounded by the x -axis, the y -axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x -axis are semicircles, what is the volume of the solid?

- (A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

38.

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

- (A) $y = 8x - 5$
 (B) $y = x + 7$
 (C) $y = x + 0.763$
 (D) $y = x - 0.122$
 (E) $y = x - 2.146$

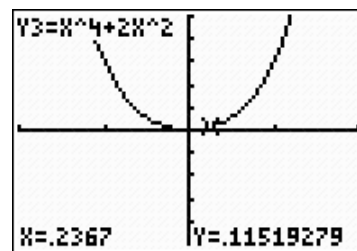
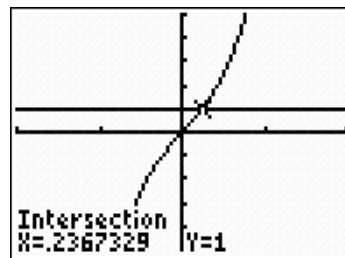
$$f'(x) = 4x^3 + 4x = 1$$

when $x = .2367$
 $y = .1152$

$$y - .1152 = 1(x - .2367)$$

$$y = x - .2367 + .1152$$

$$y = x - .122$$



1) graph $f'(x)$ and $y=1$ to find x -coordinate of intersection

2. graph $f(x)$ and find the y -value

Now:
 $m=1$
 $x=.2367$
 $y=.1152$

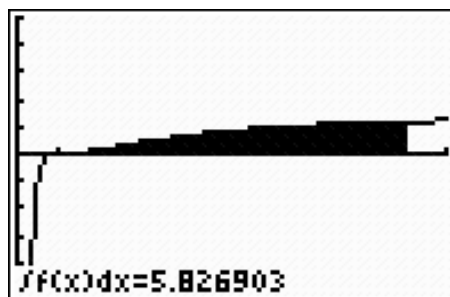
39.

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

- (A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

$$\int_1^9 \frac{(\ln x)^3}{x} dx = F(9) - F(1)$$

use calculator to find this 5.827 given



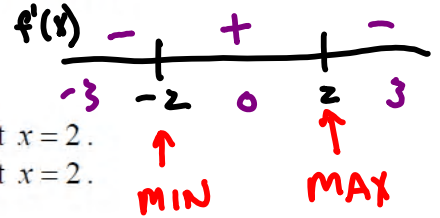
40.

always negative
↓

If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if

$$f'(x) = (x^2 - 4)g(x), \text{ which of the following is true?}$$

$$(x+2)(x-2)$$



- (A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
- (B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
- (C) f has relative minima at $x = -2$ and at $x = 2$.
- (D) f has relative maxima at $x = -2$ and at $x = 2$.
- (E) It cannot be determined if f has any relative extrema.

41.

If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

- (A) A is always increasing.
- (B) A is always decreasing.
- (C) A is decreasing only when $b < h$.
- (D) A is decreasing only when $b > h$.
- (E) A remains constant.

$$\frac{db}{dt} = 3 \text{ in/min} \quad \frac{dh}{dt} = -3 \text{ in/min}$$

$$A = \frac{1}{2}bh \quad \leftarrow \text{product rule !!}$$

$$\frac{dA}{dt} = \frac{1}{2} \left[b \frac{dh}{dt} + h \frac{db}{dt} \right]$$

$$= \frac{1}{2} [-3b + 3h]$$

$$= \frac{3}{2} [h - b]$$

positive when $h > b$
negative when $b > h$

A decreases when $b > h$

42.

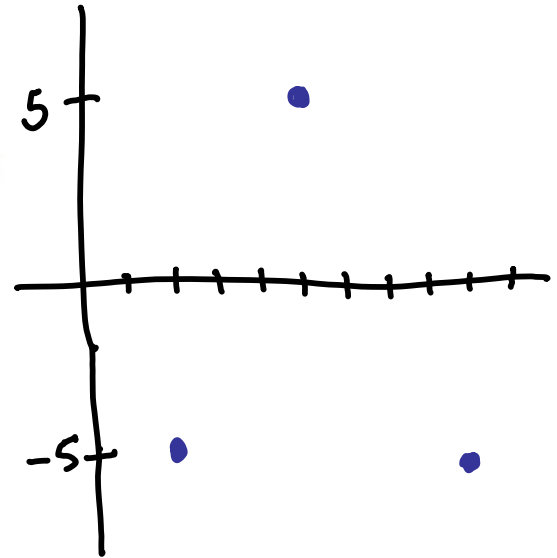
smooth curve

Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

*IVT
Rolle's Theorem
IVT*

- I. f has at least 2 zeros.
- II. The graph of f has at least one horizontal tangent.
- III. For some c , $2 < c < 5$, $f(c) = 3$.

- (A) None
- (B) I only
- (C) I and II only
- (D) I and III only
- (E) I, II, and III**



43.

If $0 \leq k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

- (A) 1.471
- (B) 1.414
- (C) 1.277
- (D) 1.120**
- (E) 0.436

$$\int_k^{\pi/2} \cos x \, dx = [\sin x]_k^{\pi/2} = \sin \frac{\pi}{2} - \sin k = 0.1$$

$$1 - \sin k = 0.1$$

$$\sin k = .9$$

or use $\sin^{-1}(.9)$

graph $\sin x$ and $y = .9$

look for pt. of intersection

on $0 \leq x < \frac{\pi}{2}$

*x min: 0
x max: $\pi/2$
y min: -.5
y max: 1
y sel: .5*

