## Question 3

An object moving along a curve in the $xy$-plane is at position $(x(t), y(t))$ at time $t$, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1+t^3}$$

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1.208$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$. $a(2) = \langle .396, -.741 \rangle$

(b) The curve has a vertical tangent line at one point. At what time $t$ is the object at this point?

b) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ when $\frac{dx}{dt} = 0$ $\frac{dy}{dx}$ is undefined and the curve has a vertical tangent

i.e. @ $t = .693$ (note: you should make sure that $\frac{dy}{dt} \neq 0$ at this value of $t$ as well)

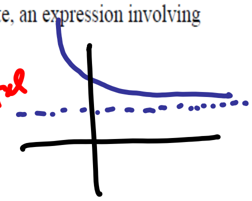
(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of $t$ and use it to evaluate $\lim_{t\to\infty} m(t)$.

$m(t) = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $\quad \lim_{t\to\infty} m(t) = \lim_{t\to\infty} \frac{dy/dt}{dx/dt} = 0$

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value $c$.

$\lim_{t\to\infty} y(t) = c$ $\quad \int_2^\infty y'(t)\,dt = y(\infty) - \cancel{y(2)}^{\,-3}$

$c = \int_2^\infty y'(t)\,dt - 3 = \lim_{t\to\infty} y(t)$ ← the horizontal asymptote



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|---|---|
| (a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740 \rangle$<br>Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or $1.208$ | $2: \begin{cases} 1: \text{acceleration} \\ 1: \text{speed} \end{cases}$ |
| (b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$<br>$1 - 2e^{-t} = 0$<br>$t = \ln 2 = 0.693$ and $\frac{dy}{dt} \neq 0$ when $t = \ln 2$ | $2: \begin{cases} 1: x'(t) = 0 \\ 1: \text{answer} \end{cases}$ |
| (c) $m(t) = \frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1-2e^{-t}\right)}$<br>$\lim_{t\to\infty} m(t) = \lim_{t\to\infty}\left(\frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1-2e^{-t}\right)}\right)$<br>$= 0\left(\frac{1}{\sin^{-1}(1)}\right) = 0$ | $2: \begin{cases} 1: m(t) \\ 1: \text{limit value} \end{cases}$ |
| (d) Since $\lim_{t\to\infty} x(t) = \infty$,<br>$c = \lim_{t\to\infty} y(t) = -3 + \int_2^\infty \frac{4t}{1+t^3}\,dt$ | $3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{initial value consistent with lower limit} \end{cases}$ |