

2 ways: If geometric then
 $|r| < 1$

Question 6

The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \dots + \frac{(-1)^n nx^n}{n+1} + \dots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \dots + \frac{(-1)^n x^n}{(2n)!} + \dots$$

for all real numbers x for which the series converges.

(a) Find the interval of convergence of the power series for f . Justify your answer.

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)x^{n+1}}{n+2} \right| \cdot \left| \frac{n+1}{nx^n} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)(n+1)}{n(n+2)} \left| \frac{x^{n+1}}{x^n} \right| = |x| < 1$$

i.e. $-1 < x < 1$

Note: when you use the ratio test to find the interval of convergence, you must always check the endpoints using one of the convergence tests.

@ $x = -1$ $\lim_{n \rightarrow \infty} \frac{(-1)^n \cdot n \cdot (-1)^n}{n+1} = \lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \neq 0$ Diverges so final interval of convergence is

@ $x = 1$ $\lim_{n \rightarrow \infty} \frac{(-1)^n \cdot n \cdot (1)^n}{n+1} = \lim_{n \rightarrow \infty} \frac{(-1)^n n}{n+1} \neq 0$ Diverges $-1 < x < 1$

(b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

if $y = f(x) - g(x)$ then $y' = f'(x) - g'(x)$ and $y'' = f''(x) - g''(x)$
 $y'(0) = f'(0) - g'(0)$ $y''(0) = f''(0) - g''(0)$ ++ --

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \dots + \frac{(-1)^n nx^n}{n+1} + \dots \quad g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \dots + \frac{(-1)^n x^n}{(2n)!} + \dots$$

$$f'(x) = -\frac{1}{2} + \frac{4x}{3} - \frac{9x^2}{4} \quad f'(0) = -\frac{1}{2} \quad g'(x) = -\frac{1}{2} + \frac{2x}{24} - \frac{3x^2}{6!} \quad g'(0) = -\frac{1}{2}$$

$$f''(x) = \frac{4}{3} - \frac{18x}{4} + \dots \quad f''(0) = \frac{4}{3} \quad g''(x) = \frac{1}{12} - \frac{6x}{6!} \quad g''(0) = \frac{1}{12}$$

$$y'(0) = f'(0) - g'(0) \quad y''(0) = f''(0) - g''(0) \quad y \text{ has a rel. min at } x=0 \text{ because } y'(0)=0 \text{ and } y''(0)>0$$

$$= -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0 \quad = \frac{4}{3} - \frac{1}{12} = \boxed{\frac{15}{12}}$$

$$(a) \left| \frac{(-1)^{n+1}(n+1)x^{n+1}}{n+2} \cdot \frac{n+1}{(-1)^n mx^n} \right| = \frac{(n+1)^2}{(n+2)(n)} \cdot |x|$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)^2}{(n+2)(n)} \cdot |x| = |x|$$

The series converges when $-1 < x < 1$.

When $x = 1$, the series is $-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \dots$

This series does not converge, because the limit of the individual terms is not zero.

When $x = -1$, the series is $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$

This series does not converge, because the limit of the individual terms is not zero.

Thus, the interval of convergence is $-1 < x < 1$.

- 5 : $\begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis/conclusion for both endpoints} \end{cases}$

$$(b) f'(x) = -\frac{1}{2} + \frac{4}{3}x - \frac{9}{4}x^2 + \dots \text{ and } f'(0) = -\frac{1}{2}.$$

$$g'(x) = -\frac{1}{2!} + \frac{2}{4!}x - \frac{3}{6!}x^2 + \dots \text{ and } g'(0) = -\frac{1}{2}.$$

$$y'(0) = f'(0) - g'(0) = 0$$

$$f''(0) = \frac{4}{3} \text{ and } g''(0) = \frac{2}{4!} = \frac{1}{12}.$$

$$\text{Thus, } y''(0) = \frac{4}{3} - \frac{1}{12} > 0.$$

Since $y'(0) = 0$ and $y''(0) > 0$, y has a relative minimum at $x = 0$.

- 4 : $\begin{cases} 1 : y'(0) \\ 1 : y''(0) \\ 1 : \text{conclusion} \\ 1 : \text{reasoning} \end{cases}$