Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2+1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1}x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

a) Speed $= 2.912$

b) $L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$

c) $\int_0^4 x'(t)\,dt = x(4) - x(0)$

$2.108 = x(4) - (-3)$

$2.108 - 3 = x(4)$

$x(4) = -.892$

d) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ at $t = 1.35766311 = 1.358$

$a(1.35766311) = \langle .135, .955 \rangle$

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$

1 : speed at $t = 4$

(b) Distance $= \displaystyle\int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$

2 : { 1 : integral; 1 : answer }

(c) $x(4) = x(0) + \displaystyle\int_0^4 x'(t)\,dt$

$= -3 + 2.10794 = -0.892$

3 : { 2 : { 1 : integrand; 1 : uses $x(0) = -3$ }; 1 : answer }

(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$.

Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle|_{t=1.35766} = \langle 0.135, 0.955\rangle$.

3 : { 1 : $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$; 1 : t-value; 1 : values for x'' and y'' }