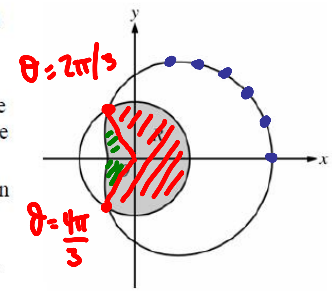
Question 3

$A = \frac{2}{3}\pi(2)^2 = \frac{8\pi}{3}$

The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.



(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$.

a) Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

$\frac{1}{2}A = \frac{1}{2}\int_{2\pi/3}^{\pi}(3+2\cos\theta)^2\,d\theta$

$A = \int_{2\pi/3}^{\pi}(3+2\cos\theta)^2\,d\theta$

Total Area $= \frac{8\pi}{3} + \int_{2\pi/3}^{\pi}(3+2\cos\theta)^2\,d\theta$

$= \frac{8\pi}{3} + 1.9928$

$= 10.370$

b) $\frac{dr}{dt} = \frac{dr}{d\theta} = \frac{d}{d\theta}[3+2\cos\theta] = -2\sin\theta$

$@\theta = \frac{\pi}{3}$, $-2\sin\theta = -2\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

Particle is moving ~~in~~ toward the origin when $\theta = \pi/3$ or $t = \pi/3$

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

$y = r\sin\theta = (3+2\cos\theta)\sin\theta = 3\sin\theta + 2\sin\theta\cos\theta = 3\sin\theta + \sin 2\theta$

$\frac{dy}{dt} = \frac{dy}{d\theta} = \frac{d}{d\theta}[3\sin\theta + \sin 2\theta] = 3\cos\theta + 2\cos 2\theta = 3\left(\frac{1}{2}\right) + 2\left(-\frac{1}{2}\right) = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

Particle is moving away from the x-axis at that point in time.

(a) Area $= \frac{2}{3}\pi(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3+2\cos\theta)^2\,d\theta$ $= 10.370$	4 : { 1 : area of circular sector; 2 : integral for section of limaçon { 1 : integrand; 1 : limits and constant }; 1 : answer }
(b) $\left.\frac{dr}{dt}\right\|_{\theta=\pi/3} = \left.\frac{dr}{d\theta}\right\|_{\theta=\pi/3} = -1.732$ The particle is moving closer to the origin, since $\frac{dr}{dt} < 0$ and $r > 0$ when $\theta = \frac{\pi}{3}$.	2 : { 1 : $\left.\frac{dr}{dt}\right\|_{\theta=\pi/3}$; 1 : interpretation }
(c) $y = r\sin\theta = (3+2\cos\theta)\sin\theta$ $\left.\frac{dy}{dt}\right\|_{\theta=\pi/3} = \left.\frac{dy}{d\theta}\right\|_{\theta=\pi/3} = 0.5$ The particle is moving away from the x-axis, since $\frac{dy}{dt} > 0$ and $y > 0$ when $\theta = \frac{\pi}{3}$.	3 : { 1 : expression for y in terms of θ; 1 : $\left.\frac{dy}{dt}\right\|_{\theta=\pi/3}$; 1: interpretation }