2008 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$. $\langle .433, -11.872 \rangle$

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

$$\int_0^4 3\cos(t^2/2)\,dt = y(4) - y(0)$$
$$3.399 = 5 - y(0)$$
$$y(0) = 5 - 3.399$$
$$y(0) = 1.601$$

(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 3.5$ for the first time in $0 \leq t \leq 4$ @ $t = 2.226$

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

Distance $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 13.182$

(a) $a(4) = \langle x''(4), y''(4) \rangle = \langle 0.433, -11.872 \rangle$

1 : answer

(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601

3 : { 1 : integrand; 1 : uses $y(4) = 5$; 1 : answer }

(c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$

$$= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$$

The particle first reaches this speed when $t = 2.225$ or 2.226.

3 : { 1 : expression for speed; 1 : equation; 1 : answer }

(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\,dt = 13.182$

2 : { 1 : integral; 1 : answer }