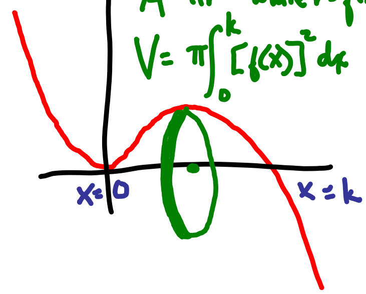
Question 4 $f(x) = kx^2 - x^3 = x^2(k-x)$

Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

$A = \pi r^2$ where $r = f(x)$

$V = \pi \int_0^k [f(x)]^2\,dx$



a) (a) Find all values of the constant k for which the area of R equals 2.

$$A = \int_0^k kx^2 - x^3\,dx = \left[\frac{kx^3}{3} - \frac{x^4}{4}\right]_0^k$$

$$= \frac{4k^4}{4\cdot 3} - \frac{k^4 \cdot 3}{4 \cdot 3} = \frac{k^4}{12} = 2 \text{ when}$$

$$k^4 = 24$$

$$k = \sqrt[4]{24}$$

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

b) $V = \pi \int_0^k (kx^2 - x^3)^2\,dx$

(c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.

$\frac{dy}{dx} = 2kx - 3x^2$

c) $P = k + \int_0^k \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

$= k + \int_0^k \sqrt{1 + (2kx - 3x^2)^2}\,dx$

(a) For $x \ge 0$, $f(x) = x^2(k-x) \ge 0$ if $0 \le x \le k$ $\int_0^k (kx^2 - x^3)\,dx = \left(\frac{k}{3}x^3 - \frac{1}{4}x^4\right)\Big\vert_{x=0}^{x=k} = \frac{k^4}{12}$ Area $= \frac{k^4}{12} = 2$; $k = \sqrt[4]{24}$	4 : { 1 : integral 1 : antiderivative 1 : value of integral 1 : answer
(b) Volume $= \pi \int_0^k (kx^2 - x^3)^2\,dx$	2 : { 1 : integrand 1 : limits and constant
(c) Perimeter $= k + \int_0^k \sqrt{1 + (2kx - 3x^2)^2}\,dx$	3 : { 1 : $\int_0^k \sqrt{1 + (f'(x))^2}\,dx$ 1 : uses $f'(x) = 2kx - 3x^2$ in integrand 1 : answer