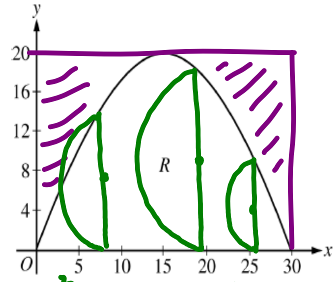
2009 SCORING GUIDELINES (Form B)

Question 1

a) $A = 600 - \int_0^{30} f(x)\,dx = 218.028\text{ cm}^2$

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



$A = \frac{\pi r^2}{2}$ where $r = \frac{f(x)}{2}$

(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

$V = \frac{\pi}{2}\int_0^{30}\left(\frac{f(x)}{2}\right)^2 dx$

$= \frac{\pi}{8}\int_0^{30}(f(x))^2\,dx = 2356.194\text{ cm}^3$

$.05 \times V = 117.810$ grams

(c) Find the perimeter of the base of the cake.

$L = 30 + \int_0^{30}\sqrt{1 + [f'(x)]^2}\,dx = 81.803$ cm

(a) Area $= 30 \cdot 20 - \int_0^{30} f(x)\,dx = 218.028\text{ cm}^2$	$3 : \begin{cases} 2 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \int_0^{30} \frac{\pi}{2}\left(\frac{f(x)}{2}\right)^2 dx = 2356.194\text{ cm}^3$ Therefore, the baker needs $2356.194 \times 0.05 = 117.809$ or 117.810 grams of chocolate.	$3 : \begin{cases} 2 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Perimeter $= 30 + \int_0^{30}\sqrt{1 + (f'(x))^2}\,dx = 81.803$ or 81.804 cm	$3 : \begin{cases} 2 : \text{integral} \\ 1 : \text{answer} \end{cases}$