$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$

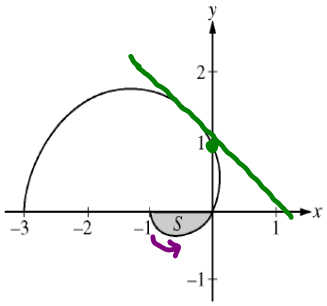
$= \frac{1}{2}\int_{\pi}^{\pi/3} (1-2\cos\theta)^2 \, d\theta$

θ	r
0	-1
$\pi/3$	0

AP® CALCULUS BC
2009 SCORING GUIDELINES (Form B)

Question 4

The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.



(a) Write an integral expression for the area of S.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

$x = r\cos\theta$

$y = r\sin\theta$

$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[(1-2\cos\theta)\cos\theta\right] = \frac{d}{d\theta}\left[\cos\theta - 2\cos^2\theta\right] = -\sin\theta + 4\cos\theta\sin\theta = -\sin\theta + 2\sin 2\theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[(1-2\cos\theta)\sin\theta\right] = \frac{d}{d\theta}\left[\sin\theta - \frac{2}{2}\sin 2\theta\right] = \cos\theta - 2\cos 2\theta$

$(0,1) \quad m = ??$ $\quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta} = \frac{0+2}{-1+0} = -2$

$y - 1 = -2(x-0)$

$y = -2x + 1$

(a) $r(0) = -1$; $r(\theta) = 0$ when $\theta = \frac{\pi}{3}$.

Area of $S = \frac{1}{2}\int_0^{\pi/3} (1-2\cos\theta)^2 \, d\theta$

$2 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \end{cases}$

(b) $x = r\cos\theta$ and $y = r\sin\theta$

$\frac{dr}{d\theta} = 2\sin\theta$

$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta = 4\sin\theta\cos\theta - \sin\theta$

$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta = 2\sin^2\theta + (1-2\cos\theta)\cos\theta$

$4 : \begin{cases} 1 : \text{uses } x = r\cos\theta \text{ and } y = r\sin\theta \\ 1 : \frac{dr}{d\theta} \\ 2 : \text{answer} \end{cases}$

(c) When $\theta = \frac{\pi}{2}$, we have $x = 0$, $y = 1$.

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{2}} = -2$

The tangent line is given by $y = 1 - 2x$.

$3 : \begin{cases} 1 : \text{values for } x \text{ and } y \\ 1 : \text{expression for } \frac{dy}{dx} \\ 1 : \text{tangent line equation} \end{cases}$