CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$.

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

WRITE ALL WORK IN THE EXAM BOOKLET.

2. For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

   (a) How many kilometers does the car travel during the first 2 hours?

   (b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

   (c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

WRITE ALL WORK IN THE EXAM BOOKLET.

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t + 10}\right)$ for $0 \le t \le 120$ minutes.

   (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

   (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

   (c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

   (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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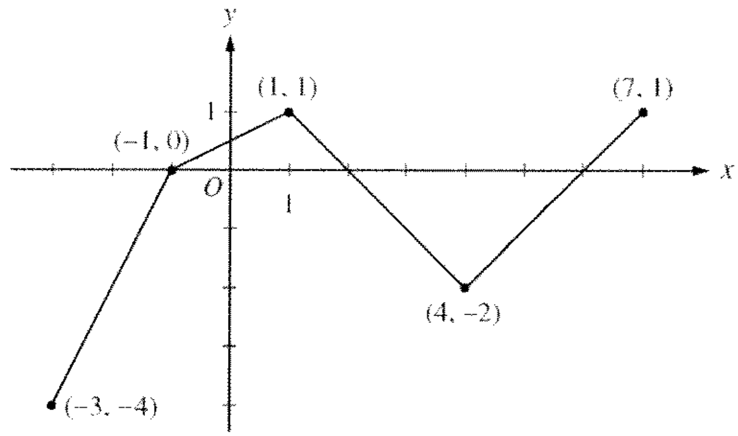
4. Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

   (a) Find all values of the constant k for which the area of R equals 2.

   (b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

   (c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.

WRITE ALL WORK IN THE EXAM BOOKLET.



Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

   (a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

   (b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

   (c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

   (d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

WRITE ALL WORK IN THE EXAM BOOKLET.

6. Let f be the function given by $f(x) = \frac{2x}{1 + x^2}$.

   (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

   (b) Does the series found in part (a), when evaluated at $x = 1$, converge to $f(1)$? Explain why or why not.

   (c) The derivative of $\ln(1 + x^2)$ is $\frac{2x}{1 + x^2}$. Write the first four nonzero terms of the Taylor series for $\ln(1 + x^2)$ about $x = 0$.

   (d) Use the series found in part (c) to find a rational number A such that $\left|A - \ln\left(\frac{5}{4}\right)\right| < \frac{1}{100}$. Justify your answer.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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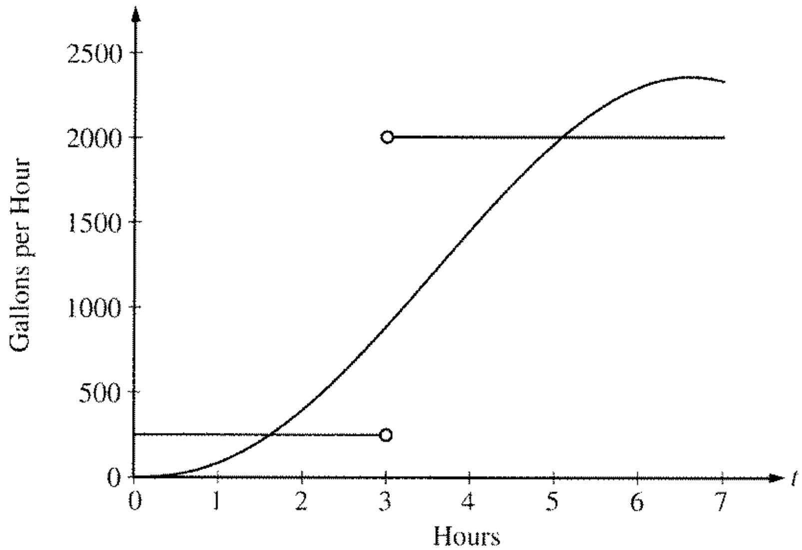
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

   (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

   (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

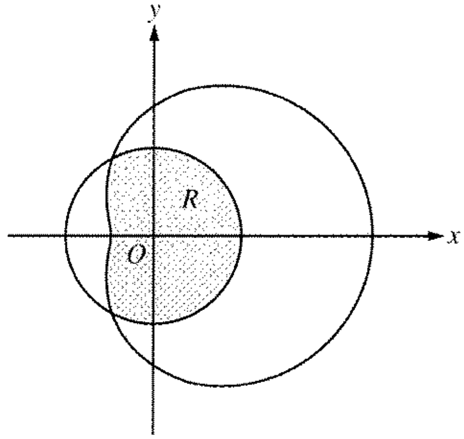
The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



3. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

   (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

   (b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

   (c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. Let f be the function defined for $x > 0$, with $f(e) = 2$ and f', the first derivative of f, given by $f'(x) = x^2 \ln x$.

(a) Write an equation for the line tangent to the graph of f at the point $(e, 2)$.

(b) Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.

(c) Use antidifferentiation to find $f(x)$.

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t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

6. Let f be the function given by $f(x) = e^{-x^2}$.

   (a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

   (b) Use your answer to part (a) to find $\lim_{x \to 0} \frac{1 - x^2 - f(x)}{x^4}$.

   (c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}\,dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}\,dt$.

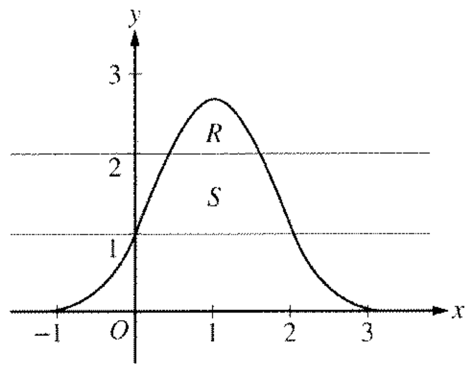
   (d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2}\,dt$ by less than $\frac{1}{200}$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

WRITE ALL WORK IN THE EXAM BOOKLET.

2. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2 + 1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

WRITE ALL WORK IN THE EXAM BOOKLET.

3. The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

   (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

   (b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

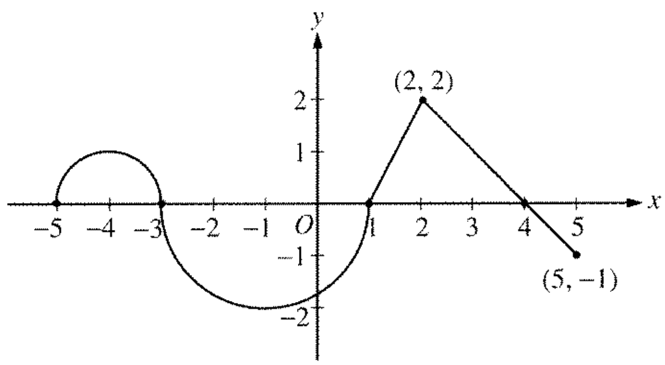
   (c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.
   (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
   (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
   (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
   (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Find the values of the constants m, b, and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.

(d) Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k.

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6. Let f be the function given by $f(x) = 6e^{-x/3}$ for all x.

(a) Find the first four nonzero terms and the general term for the Taylor series for f about $x = 0$.

(b) Let g be the function given by $g(x) = \int_0^x f(t)\,dt$. Find the first four nonzero terms and the general term for the Taylor series for g about $x = 0$.

(c) The function h satisfies $h(x) = kf'(ax)$ for all x, where a and k are constants. The Taylor series for h about $x = 0$ is given by

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots.$$

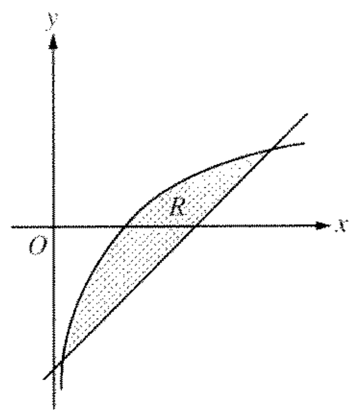
Find the values of a and k.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.
   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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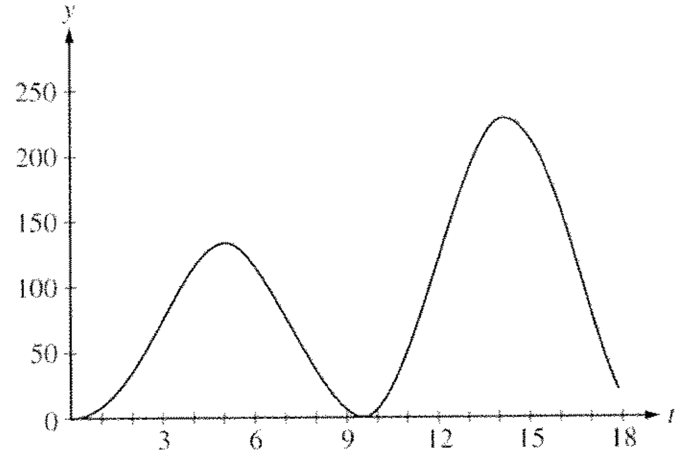

2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

3. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim\limits_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

   (a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

   (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

5. Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

   (a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.

   (b) Is it possible for the x-axis to be tangent to the graph of f at some point? Explain why or why not.

   (c) Find the second-degree Taylor polynomial for f about $x = -1$.

   (d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

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6. The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.

   (a) Find the interval of convergence of the power series for f. Justify your answer.

   (b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM