

1973 AB

1. $\int (x^3 - 3x) dx =$

(A) $3x^2 - 3 + C$

(B) $4x^4 - 6x^2 + C$

(C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$

(E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$

(B) $\frac{7}{3}$

(C) 3

(D) 7

(E) 8

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t , what is $s(4) - s(2)$?

(A) 20

(B) 24

(C) 28

(D) 32

(E) 42

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$

(B) $e - 1$

(C) $2(e - 1)$

(D) $2e - 1$

(E) $2e$

17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$

(B) $\frac{7}{3}$

(C) $\frac{9}{2}$

(D) $\frac{31}{6}$

(E) $\frac{33}{2}$

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x , then $\int_a^b f(x) dx$ is

- (A) $F'(a) - F'(b)$
- (B) $F'(b) - F'(a)$
- (C) $F(a) - F(b)$
- (D) $F(b) - F(a)$
- (E) none of the above

21. $\int_0^1 (x+1)e^{x^2+2x} dx =$

- (A) $\frac{e^3}{2}$
- (B) $\frac{e^3-1}{2}$
- (C) $\frac{e^4-e}{2}$
- (D) $e^3 - 1$
- (E) $e^4 - e$

25. $\int_0^{\pi/4} \tan^2 x dx =$

- (A) $\frac{\pi}{4} - 1$
- (B) $1 - \frac{\pi}{4}$
- (C) $\frac{1}{3}$
- (D) $\sqrt{2} - 1$
- (E) $\frac{\pi}{4} + 1$

27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}} dx =$

- (A) $1 - \frac{\sqrt{3}}{2}$
- (B) $\frac{1}{2} \ln \frac{3}{4}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{6} - 1$
- (E) $2 - \sqrt{3}$

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

- (A) 1
- (B) $\frac{4}{3}$
- (C) $\frac{5}{3}$
- (D) 2
- (E) 5

30. $\int_1^2 \frac{x-4}{x^2} dx =$

- (A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

32. $\int \frac{5}{1+x^2} dx =$

- (A) $\frac{-10x}{(1+x^2)^2} + C$ (B) $\frac{5}{2x} \ln(1+x^2) + C$ (C) $5x - \frac{5}{x} + C$
(D) $5 \arctan x + C$ (E) $5 \ln(1+x^2) + C$

34. The average value of \sqrt{x} over the interval $0 \leq x \leq 2$ is

- (A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

38. If $\int_1^2 f(x-c) dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x) dx =$

- (A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5

41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^1 f(x) dx =$

- (A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$

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2. $\int_0^3 (x+1)^{1/2} dx =$

- (A) $\frac{21}{2}$ (B) 7 (C) $\frac{16}{3}$ (D) $\frac{14}{3}$ (E) $-\frac{1}{4}$

5. $\int_{-1}^2 \frac{|x|}{x} dx$ is

- (A) -3 (B) 1 (C) 2 (D) 3 (E) nonexistent

12. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1} dx = \frac{1}{n}$?

- (A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$
(D) $2^{1/n}$ (E) 2^n

20. $\int x\sqrt{4-x^2} dx =$

- (A) $\frac{(4-x^2)^{3/2}}{3} + C$ (B) $-(4-x^2)^{3/2} + C$ (C) $\frac{x^2(4-x^2)^{3/2}}{3} + C$
(D) $-\frac{x^2(4-x^2)^{3/2}}{3} + C$ (E) $-\frac{(4-x^2)^{3/2}}{3} + C$

28. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}} dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y dy$ (B) $2 \int_0^{1/2} \frac{\sin^2 y}{\cos y} dy$ (C) $2 \int_0^{\pi/4} \sin^2 y dy$

(D) $\int_0^{\pi/4} \sin^2 y dy$ (E) $2 \int_0^{\pi/6} \sin^2 y dy$