AP Calculus Basic Integration Free Response Problem Set                    Name:

1.

(a) Given $5x^3 + 40 = \int_c^x f(t)\,dt$.

(i) Find $f(x)$.

(ii) Find the value of $c$.

(b) If $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt$, find $F'(x)$.

2.

Let $f$ be a continuous function that is defined for all real numbers $x$ and that has the following properties.

(i) $\int_1^3 f(x)\,dx = \frac{5}{2}$ (ii) $\int_1^5 f(x)\,dx = 10$

(a) Find the average (mean) value of $f$ over the closed interval $[1, 3]$.

(b) Find the value of $\int_3^5 (2f(x) + 6)\,dx$.

(c) Given that $f(x) = ax + b$, find the values of $a$ and $b$.

3.

A particle moves along the $x$-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is –9, and its position is –27.

(a) Find $v(t)$, the velocity of the particle at any time $t \geq 0$.

(b) For what values of $t \geq 0$ is the particle moving to the right?

(c) Find $x(t)$, the position of the particle at any time $t \geq 0$.

4.

A particle moves along the $x$-axis with acceleration given by $a(t) = \cos t$ for $t \geq 0$. At $t = 0$, the velocity $v(t)$ of the particle is 2, and the position $x(t)$ is 5.

(a) Write an expression for the velocity $v(t)$ of the particle.

(b) Write an expression for the position $x(t)$.

(c) For what values of $t$ is the particle moving to the right? Justify your answer.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

5.

A particle moves along the $x$-axis so that its acceleration at any time $t$ is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time $t$.

(b) For what values of $t$ is the particle at rest?

(c) Write an expression for the position $x(t)$ of the particle at any time $t$.

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

6.

Let $f$ be a differentiable function, defined for all real numbers $x$, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)\,dx = 18$

Find $f(x)$. Show your work.

7.

Let $f$ be a function such that $f''(x) = 6x + 8$.

(a) Find $f(x)$ if the graph of $f$ is tangent to the line $3x - y = 2$ at the point $(0, -2)$.

(b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

8.

A particle, initially at rest, moves along the $x$-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

(a) Find the values of $t$ for which the particle is at rest.

(b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

9.

Let $f$ be the function that is defined for all real numbers $x$ and that has the following properties.

(i) $f''(x) = 24x - 18$

(ii) $f'(1) = -6$

(iii) $f(2) = 0$

(a) Find each $x$ such that the line tangent to the graph of $f$ at $(x, f(x))$ is horizontal.

(b) Write an expression for $f(x)$.

(c) Find the average value of $f$ on the interval $1 \le x \le 3$.

10.

A particle moves on the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of $t$ for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

11.

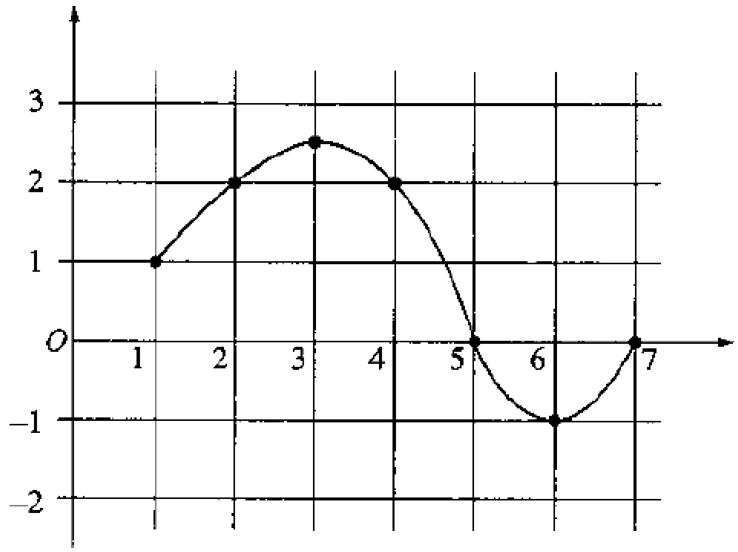
A particle moves along the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

(b) For what values of $t$, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0,3]$?

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

12.




The graph of a differentiable function $f$ on the closed interval $[1,7]$ is shown above.

Let $h(x) = \int_1^x f(t)\,dt$ for $1 \le x \le 7$.

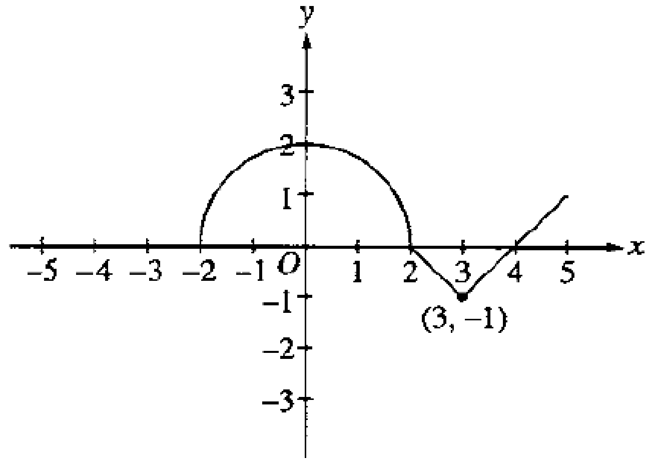
(a) Find $h(1)$ .

(b) Find $h'(4)$ .

(c) On what interval or intervals is the graph of $h$ concave upward? Justify your answer.

(d) Find the value of $x$ at which $h$ has its minimum on the closed interval $[1,7]$. Justify your answer.

14.




The graph of the function $f$ consists of a semicircle and two line segments as shown above. Let $g$ be the function given by $g(x)=\int_0^x f(t)dt$ .

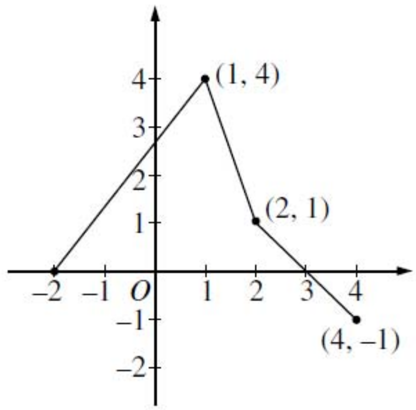
(a) Find $g(3)$ .

(b) Find all the values of $x$ on the open interval $(-2,5)$ at which $g$ has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of $g$ at $x=3$ .

(d) Find the $x$-coordinate of each point of inflection of the graph of $g$ on the open interval $(-2,5)$ . Justify your answer.

15.




The graph of the function $f$, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

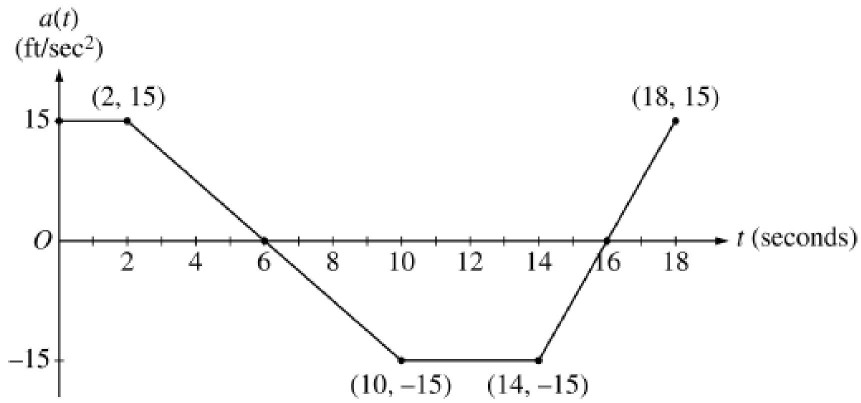
(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of $g$, with respect to $x$, at $x = 1$.

(c) Find the absolute minimum value of $g$ on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of $g$ is not defined at $x = 1$ and $x = 2$. How many of these values are $x$-coordinates of points of inflection of the graph of $g$? Justify your answer.

16.




A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec$^2$, is the piecewise linear function defined by the graph above.

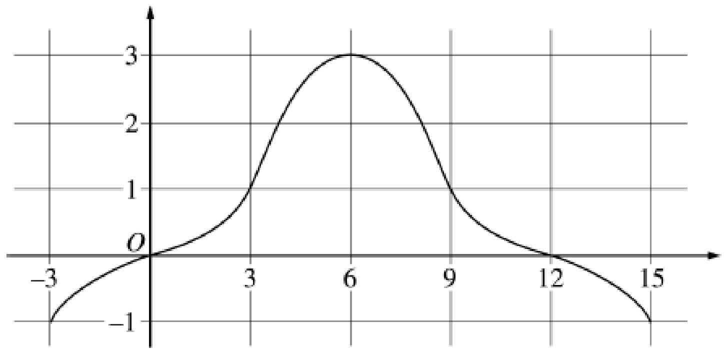
(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

17.




Graph of $f$

The graph of a differentiable function $f$ on the closed interval $[-3, 15]$ is shown in the figure above. The graph of $f$ has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

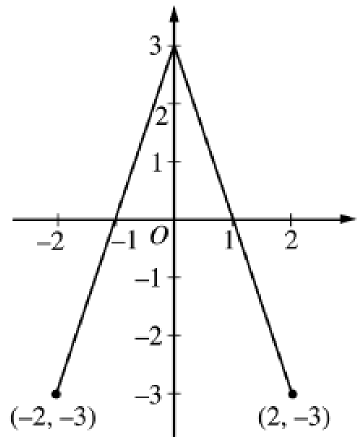
(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is $g$ decreasing? Justify your answer.

(c) On what intervals is the graph of $g$ concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

18.




Graph of $f$

The graph of the function $f$ shown above consists of two line segments. Let $g$ be the function given by $g(x) = \int_0^x f(t)\, dt$.

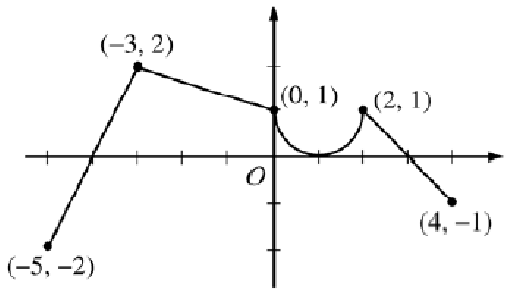
(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of $x$ in the open interval $(-2, 2)$ is $g$ increasing? Explain your reasoning.

(c) For what values of $x$ in the open interval $(-2, 2)$ is the graph of $g$ concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of $g$ on the closed interval $[-2, 2]$.
**(Note: The axes are provided in the pink test booklet only.)**

19.



Graph of $f$

The graph of the function $f$ shown above consists of a semicircle and three line segments. Let $g$ be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

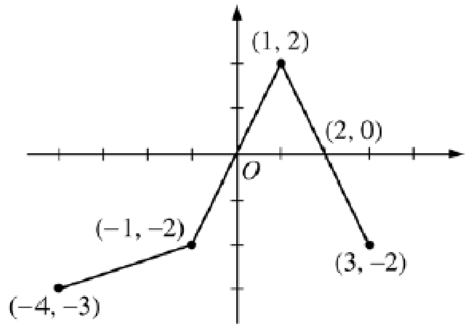
(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of $x$ in the open interval $(-5, 4)$ at which $g$ attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of $g$ on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of $x$ in the open interval $(-5, 4)$ at which the graph of $g$ has a point of inflection.

20.



Graph of $f$

The graph of the function $f$ above consists of three line segments.

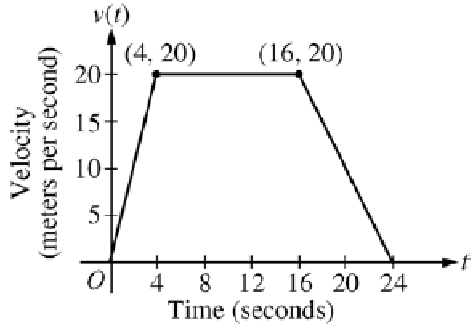
(a) Let $g$ be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function $g$ defined in part (a), find the $x$-coordinate of each point of inflection of the graph of $g$ on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let $h$ be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of $x$ in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function $h$ defined in part (c), find all intervals on which $h$ is decreasing. Explain your reasoning.

21.




A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time $t$, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of $v$ over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of $c$, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?