Limits, Continuity, Differentiation

If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if $f$ is continuous at $x = 2$, then $k =$

(A) $0$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $1$ (E) $\frac{7}{5}$

What is $\lim\limits_{h \to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) $0$ (B) $\frac{1}{2}$ (C) $1$ (D) The limit does not exist.

(E) It cannot be determined from the information given.

$\lim\limits_{n \to \infty} \dfrac{4n^2}{n^2 + 10{,}000n}$ is

(A) $0$ (B) $\frac{1}{2{,}500}$ (C) $1$ (D) $4$ (E) nonexistent

If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim\limits_{h \to 0} \dfrac{e^{x+h}}{h}$ (B) $\lim\limits_{h \to 0} \dfrac{e^{x+h} - e^e}{h}$ (C) $\lim\limits_{h \to 0} \dfrac{e^{e+h} - e}{h}$

(D) $\lim\limits_{h \to 0} \dfrac{e^{x+h} - 1}{h}$ (E) $\lim\limits_{h \to 0} \dfrac{e^{e+h} - e^e}{h}$

$\lim_{x\to 0}(x\csc x)$ is

(A) $-\infty$ (B) $-1$ (C) $0$ (D) $1$ (E) $\infty$

At $x = 3$, the function given by $f(x)=\begin{cases} x^2, & x<3 \\ 6x-9, & x \geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

The $\lim_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) $0$ (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

If $\lim_{x\to 3} f(x) = 7$, which of the following must be true?

I. $f$ is continuous at $x = 3$.
II. $f$ is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None (B) II only (C) III only

(D) I and III only (E) I, II, and III

$\lim_{n\to\infty} \dfrac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) $-5$ (B) $-2$ (C) $1$ (D) $3$ (E) nonexistent

If the function $f$ is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) $-4$ (B) $-2$ (C) $-1$ (D) $0$ (E) $2$

$\lim_{\theta\to 0} \dfrac{1 - \cos\theta}{2\sin^2\theta}$ is

(A) $0$ (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$ (D) $1$ (E) nonexistent

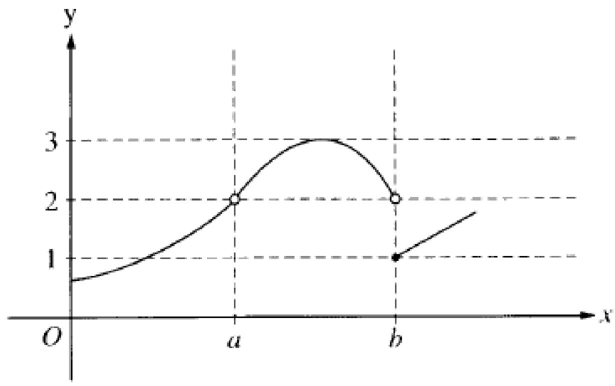
If $f$ is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

II. $\lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$

III. $\lim_{x\to a} \dfrac{f(x+h) - f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III




The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

$\lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $e$ (E) nonexistent

Let $f$ be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. $f$ is continuous at $x = 2$.

II. $f$ is differentiable at $x = 2$.

III. The derivative of $f$ is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent