Let f be the function given by $f(x) = 2\ln(x^2+3) - x$ with domain $-3 \le x \le 5$.

(a) Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

(b) Find the x-coordinate of each inflection point of f.

(c) Find the absolute maximum value of $f(x)$.

$$f'(x) = \frac{2}{x^2+3}(2x) - 1 = \frac{4x}{x^2+3} - \frac{x^2+3}{x^2+3}$$

$$= \frac{-x^2+4x-3}{x^2+3} = \frac{-(x^2-4x+3)}{x^2+3}$$

$$= \frac{-(x-3)(x-1)}{x^2+3} \qquad x=1, \; x=3$$

$-3 \quad - \quad 0 \quad 1 \quad + \quad 2 \quad 3 \quad - \quad 4 \quad 5$

Rel. min when $f'(x)$ changes from $-$ to $+$
i.e. @ $x = 1$

Rel. max when $f'(x)$ changes from $+$ to $-$
i.e. @ $x = 3$

$$f''(x) = \frac{(x^2+3)(-2x+4) - (-x^2+4x-3)(2x)}{(x^2+3)^2}$$

$$= \frac{-\cancel{2x^3} + 4x^2 - \cancel{6x} + 12 + \cancel{2x^3} - 8x^2 + \cancel{6x}}{(x^2+3)^2}$$

$$= \frac{-4x^2+12}{(x^2+3)^2} = \frac{-4(x^2-3)}{(x^2+3)^2} = 0 \text{ when } x = \pm\sqrt{3}$$

$$x = \frac{-3 \pm \sqrt{9-4(1)(3)}}{2} = \frac{-3 \pm \sqrt{21}}{2}$$

$-3 \quad - \quad -2 \quad -\sqrt{3} \quad + \quad 0 \quad \sqrt{3} \quad - \quad 2 \quad 5$

Point of inflection when $f''(x)$ changes sign
i.e. @ $x = \pm\sqrt{3}$

Abs. max at critical values or end points

x	$f(x) = 2\ln(x^2+3) - x$
-3	$2\ln 12 + 3 \approx 7.97$ ✓ ABS MAX
1	$2\ln 4 - 1 \approx 1.77$
3	$2\ln 12 - 3 \approx 1.97$
5	$2\ln 28 - 5 \approx 1.66$

Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

a) $\lim\limits_{x \to \infty} \dfrac{2x}{e^x} = 0$

$\lim\limits_{x \to -\infty} \dfrac{2x}{e^x} = \dfrac{-\infty}{0} = -\infty$

(a) Write an equation of the horizontal asymptote for the graph of f.

(b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

(c) For what values of x is the graph of f concave down?

(d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ on the axes provided below.

b)

$f(x) = 2xe^{-x}$

$f'(x) = 2x(-e^{-x}) + e^{-x} \cdot 2$

$= -2e^{-x}[x-1]$

$= 0$ when $x = 1$

critical pts. at $x = 1$ (rel. max)

$f'(x)$ $+$ | $-$

0 1 2

c) $f'(x) = 2e^{-x} - 2xe^{-x}$

$f''(x) = -2e^{-x} - (2e^{-x} - 2xe^{-x})$

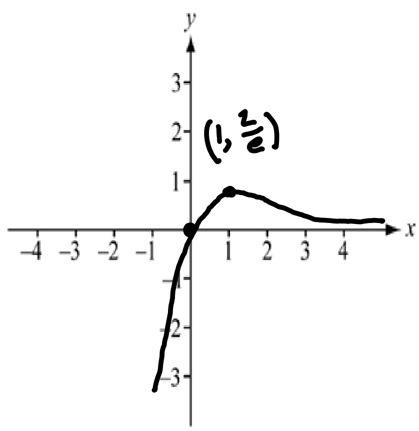
$= -4e^{-x} + 2xe^{-x}$

$= -2e^{-x}[2-x]$

$f''(x)$ $-$ | $+$

0 2 3

$f(x)$ concave down when $f''(x) < 0$; i.e. $x < 2$



Let f be the function given by $f(x) = x^3 - 7x + 6$.

a)
$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

$(x-1)(x^2-x-6)$

(a) Find the zeros of f.

$(x-1)(x-3)(x+2)$

$x=1$ $x=3$ $x=-2$

(b) Write an equation of the line tangent to the graph of f at $x = -1$.

$f'(x) = 3x^2 - 7$ $f'(-1) = 3-7 = -4$ $f(-1) = -1+7+6 = 12$

(c) Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1,3]$.

$y - y_0 = m(x - x_0)$

$y - 12 = -4(x+1)$

$y - 12 = -4x - 4$

$\underline{\underline{y = -4x + 8}}$

$$\frac{f(3)-f(1)}{3-1} = \frac{27-21+6-(1-7+6)}{2}$$

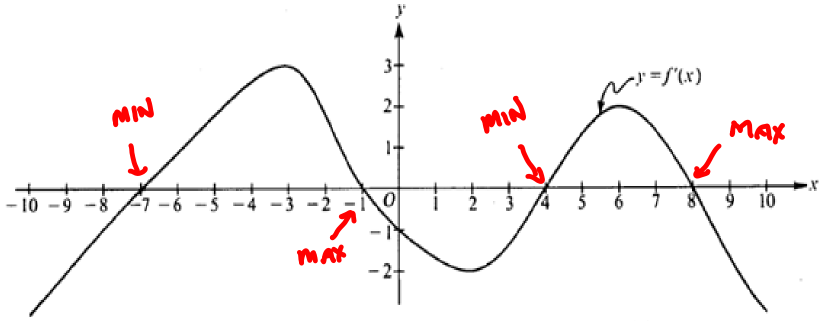
$$= \frac{12-0}{2}$$

$$= 6$$

$3x^2 - 7 = 6$

$3x^2 - 13 = 0$

$x^2 = \frac{13}{3}$ $x = \pm\sqrt{\frac{13}{3}}$, but only $\sqrt{\frac{13}{3}}$ is in the interval $[1,3]$



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

when $f'(x)=0$, i.e. @ $x=-7$ $x=-1$ $x=4$ $x=8$

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer. when $f'(x)$ changes from + to −, i.e. at $x=-1$, $x=8$

(c) For value of x is the graph of f concave downward?

when $f'(x)$ is decreasing, i.e. on $(-3,2) \cup (6,10)$

Let f be a function that is even and continuous on the closed interval $[-3, 3]$. The function f and its derivatives have the properties indicated in the table below.

if f(x) is continuous but f'(x) DNE, then either a sharp turn or vertical tangent

When a function is even, it is reflected over the x-axis

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	Positive +	0	Negative −	−1	Negative −
$f'(x)$	Undefined SHARP TURN	Negative DECR	0	Negative DECR	Undefined SHARP TURN	Positive INCR
$f''(x)$	Undefined	Positive ∪	0	Negative ∩	Undefined	Negative ∩

with this problem, I would graph first, then answer the questions.

(a) Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each x-coordinate you give, state whether f attains an absolute maximum or an absolute minimum.

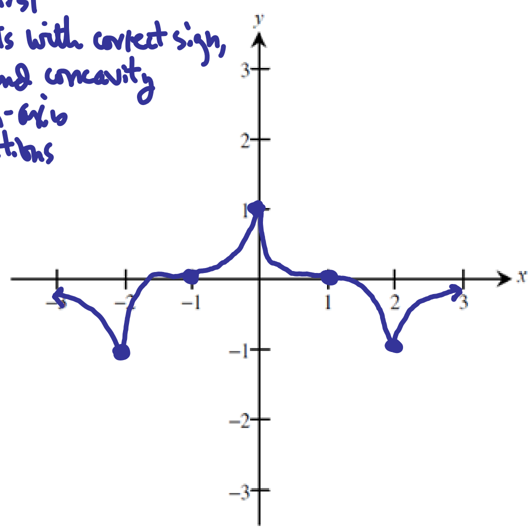
ABS. MAX: $X=0$ ABS. MIN: $X=-2, X=2$

(b) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.

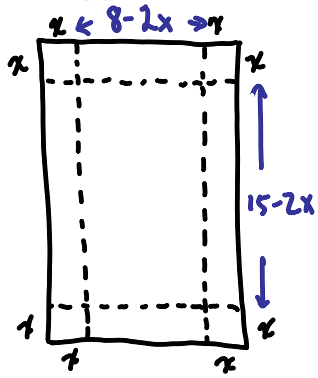
when $f''(x)$ changes sign, @ $X=-1$, $x=1$

(c) In the xy-plane provided below, sketch the graph of a function with all the given characteristics of f.

1. plot points first
2. connect points with correct sign, incr/decr, and concavity
3. reflect over X-axis
4. answer questions



Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.



$$V = (15-2x)(8-2x)x$$
$$= (120-30x-16x+4x^2)x$$
$$= 120x-30x^2-16x^2+4x^3$$
$$= 4x^3-46x^2+120x$$
$$V' = 12x^2-92x+120$$
$$= 4(3x^2-23x+30)$$
$$= 4(3x^2-18x-5x+30)$$
$$= 4[3x(x-6)-5(x-6)]$$
$$= 4(3x-5)(x-6)$$
$= 0$ when $3x-5=0$ or $x-6=0$

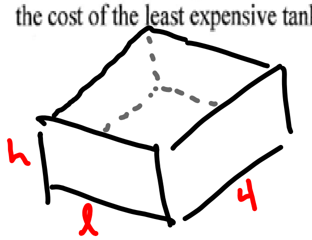
$x = 5/3$ $\quad$ $x = 6$ X

but $0 < x < 4$

23	0
22	1
21	2
20	3
19	4
18	5

$$V\left(\tfrac{5}{3}\right) = \left(15-\tfrac{10}{3}\right)\left(8-\tfrac{10}{3}\right)\tfrac{5}{3}$$
$$= \left(\tfrac{35}{3}\right)\left(\tfrac{14}{3}\right)\left(\tfrac{5}{3}\right)$$
$$= \frac{2450}{27}$$
$$= 90.74$$

A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank?



$V = l \times w \times h = 36$ $w = 4$

$4lh = 36$

$lh = 9$

$l = 9/h$ or $h = 9/l$

Area of Sides: $2lh + 2(4h)$

$2lh + 8h$

Area of Base: $4l$

Cost of Base: $10(4l) = 40l$

Cost of Sides: $5(2lh + 8h)$

$10lh + 40h$

Total cost: $\underbrace{10lh + 40h}_{\text{sides}} + \underbrace{40l}_{\text{base}}$ let $l = 9/h$

$= 10\left(\frac{9}{h}\right)h + 40h + 40\left(9/h\right)$

$C(h) = 90 + 40h + \frac{360}{h}$

To find the least expensive tank, find $C'(h)$ and determine where it changes from $-$ to $+$.

$C'(h) = 40 + 360\left(-\frac{1}{h^2}\right) = 40 - \frac{360}{h^2} = 0$ when $40h^2 = 360$

$h^2 = 9$

$h = 3$

$h = 3$ $l = 9/h = 9/3 = 3$ $w = 4$

$C(3) = 90 + 40(3) + \frac{360}{3}$

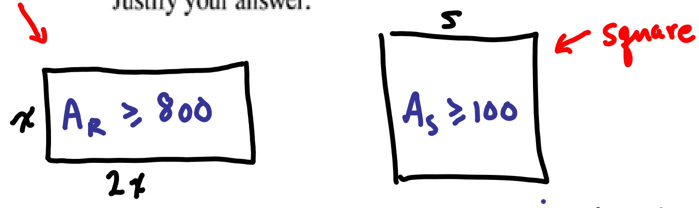
$= 90 + 120 + 120$

$= \underline{\underline{330}}$

A man has 340 yard of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square yards and the rectangular one must contain at least 800 square yards.

(a) If x is the width of the rectangular field, what are the maximum and minimum possible values of x?

(b) What is the greatest number of square yards that can be enclosed in the two fields? Justify your answer.



Perimeter of Rectangle : $P_R = 6x$

Area of rectangle : $A_R = 2x^2$

Perimeter of Square : $P_s = 340 - 6x$

Side of square : $S = \frac{340-6x}{4} = \frac{170-3x}{2}$

Area of square : $A_s = \left(\frac{170-3x}{2}\right)^2$

So x must satisfy:

$$2x^2 \geq 800$$
$$x^2 \geq 400$$
$$\boxed{x \geq 20}$$

and

$$\left(\frac{170-3x}{2}\right)^2 \geq 100$$
$$\frac{170-3x}{2} \geq 10$$
$$170 - 3x \geq 20$$
$$150 \geq 3x$$
$$50 \geq x \text{ or } \boxed{x \leq 50}$$

$$\therefore \quad \boxed{20 \leq x \leq 50}$$

Total Area: $A_T = 2x^2 + \left(\frac{170-3x}{2}\right)^2 = 2x^2 + \frac{1}{4}(170-3x)^2$

To find max area, find A_T' and determine where it changes from + to −.

$$A_T' = 4x + \frac{1}{2}(170-3x)(-3)$$
$$= 4x - \frac{3}{2}(170) + \frac{9}{2}x$$
$$= \frac{17}{2}x - 255 = 0 \text{ when } \frac{17}{2}x = 255$$
$$17x = 510$$
$$\boxed{x = 30}$$

$$A_T = A_s + A_R$$
$$= \left(\frac{170-3x}{2}\right)^2 + 2x^2$$
$$= \left(\frac{170-3(30)}{2}\right)^2 + 2(30)^2$$
$$= \left(\frac{80}{2}\right)^2 + 2(900)$$
$$= 1600 + 1800 = 3400 \text{ ft}^2$$

A particle moves on the x-axis in such a way that its position at time t is given by $x(t) = (2t-1)(t-1)^2$.

(a) At what times t is the particle at rest? Justify your answer.

At rest when $x'(t) = 0$, i.e. $t = 2/3$, $t = 1$

(b) During what interval of time is the particle moving to the left? Justify your answer.

Moving left when $x'(t) < 0$, i.e. $2/3 < t < 1$

(c) At what time during the interval found in part (b) is the particle moving most rapidly (that is, the speed is a maximum)? Justify your answer.

when $x''(t)$ changes sign on $2/3 < t < 1$, i.e. @ $t = 5/6$

$$x'(t) = (2t-1)\frac{d}{dt}\left[(t-1)^2\right] + (t-1)^2\frac{d}{dt}\left[2t-1\right]$$

(a)

$$= (2t-1)\cdot 2(t-1)^1(1) + (t-1)^2(2)$$

$$= (2t-1)(2t-2) + 2\left[t^2-2t+1\right]$$

$$= 4t^2 - 4t - 2t + 2 + 2t^2 - 4t + 2$$

$$= 6t^2 - 10t + 4$$

$$= 2(3t^2 - 5t + 2)$$

$$= 2(3t-2)(t-1)$$

$= 0$ when $3t - 2 = 0$, $t - 1 = 0$

$t = 2/3$, $t = 1$

(b)

$x'(t)$: $+$ | $-$ | $+$

$1/2$ $2/3$ $3/4$ 1 2

plug into factored form

(c) find where $x''(t)$ changes sign on $2/3 < t < 1$

$x''(t) = 12t - 10 = 2(6t-5) = 0$ when $t = 5/6$

$x''(t)$: $-$ | $+$

$2/3$ $5/6$ 1