

Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.

2

A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank?

A man has 340 yard of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square yards and the rectangular one must contain at least 800 square yards.

- (a) If  $x$  is the width of the rectangular field, what are the maximum and minimum possible values of  $x$ ?
- (b) What is the greatest number of square yards that can be enclosed in the two fields? Justify your answer.

4

A particle moves on the  $x$ -axis in such a way that its position at time  $t$  is given by  $x(t) = (2t-1)(t-1)^2$ .

- (a) At what times  $t$  is the particle at rest? Justify your answer.
- (b) During what interval of time is the particle moving to the left? Justify your answer.
- (c) At what time during the interval found in part (b) is the particle moving most rapidly (that is, the speed is a maximum)? Justify your answer.

5

A particle moves along the  $x$ -axis in such a way that its position at time  $t$  is given by  $x = 3t^4 - 16t^3 + 24t^2$  for  $-5 \leq t \leq 5$ .

- (a) Determine the velocity and acceleration of the particle at time  $t$ .
- (b) At what values of  $t$  is the particle at rest?
- (c) At what values of  $t$  does the particle change direction?
- (d) What is the velocity when the acceleration is first zero?

6

A particle moves along the  $x$ -axis in such a way that its position at time  $t$  for  $t \geq 0$  is given by  $x = \frac{1}{3}t^3 - 3t^2 + 8t$ .

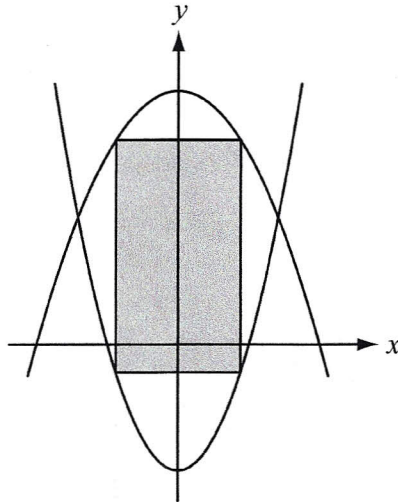
- (a) Show that at time  $t = 0$ , the particle is moving to the right.
- (b) Find all values of  $t$  for which the particle is moving to the left.
- (c) What is the position of the particle at time  $t = 3$ ?
- (d) When  $t = 3$ , what is the total distance the particle has traveled?

7

Consider  $f(x) = \cos^2 x + 2 \cos x$  over one complete period beginning with  $x = 0$ .

- (a) Find all values of  $x$  in this period at which  $f(x) = 0$ .
- (b) Find all values of  $x$  in this period at which the function has a minimum. Justify your answer.
- (c) Over what intervals in this period is the curve concave up?

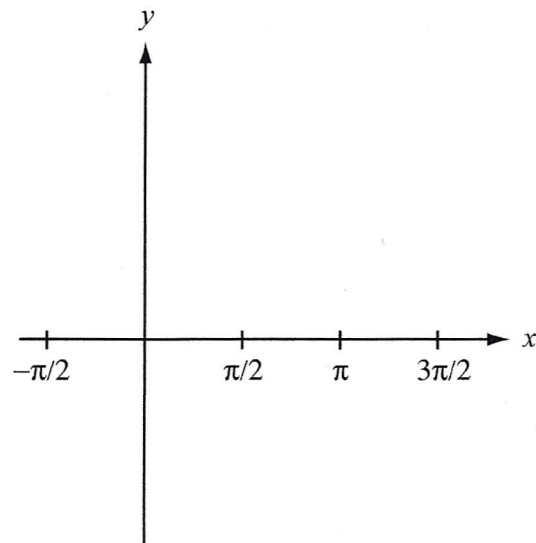
Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of  $f(x) = 18 - x^2$  and  $g(x) = 2x^2 - 9$ .





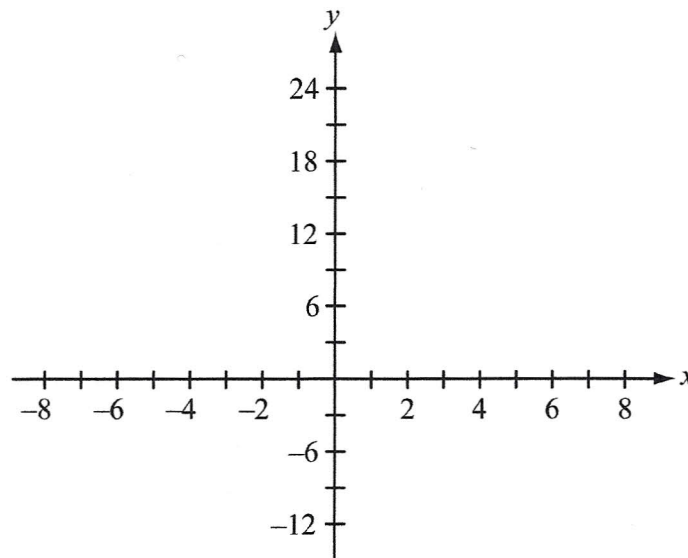
Given the function defined by  $y = x + \sin x$  for all  $x$  such that  $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ .

- (a) Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.
- (b) Find the coordinates of all points of inflection on the given interval. Justify your answers.
- (c) On the axes provided, sketch the graph of the function.



Consider the function  $f$  given by  $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$  on the interval  $-8 \leq x \leq 8$ .

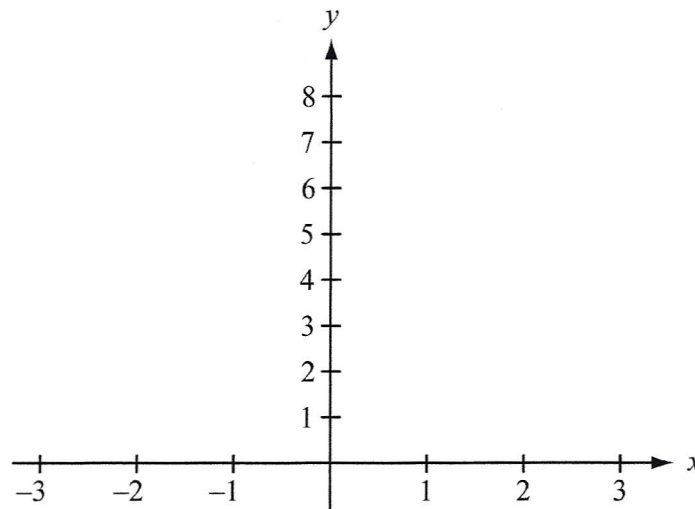
- Find the coordinates of all points at which the tangent to the curve is a horizontal line.
- Find the coordinates of all points at which the tangent to the curve is a vertical line.
- Find the coordinates of all points at which the absolute maximum and absolute minimum occur.
- For what values of  $x$  is this function concave down?
- On the axes provided, sketch the graph of the function on this interval.



A function  $f$  is continuous on the closed interval  $[-3, 3]$  such that  $f(-3) = 4$  and  $f(3) = 1$ . The functions  $f'$  and  $f''$  have the properties given in the table below.

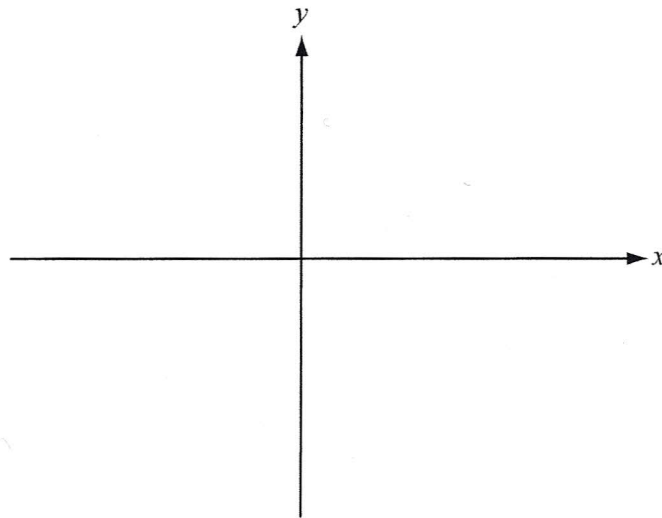
$x$	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

- (a) What are the  $x$ -coordinates of all absolute maximum and absolute minimum points of  $f$  on the interval  $[-3, 3]$ ? Justify your answer.
- (b) What are the  $x$ -coordinates of all points of inflection of  $f$  on the interval  $[-3, 3]$ ? Justify your answer.
- (c) On the axes provided, sketch a graph that satisfies the given properties of  $f$ .



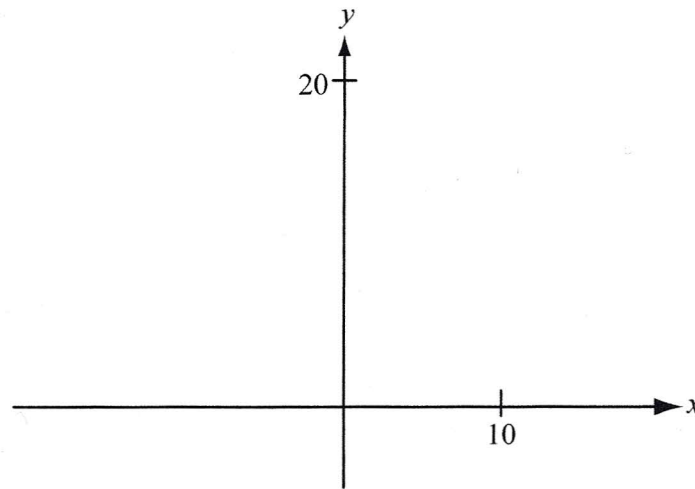
Consider the function  $f$  defined by  $f(x) = (x^2 - 1)^3$  for all real numbers  $x$ .

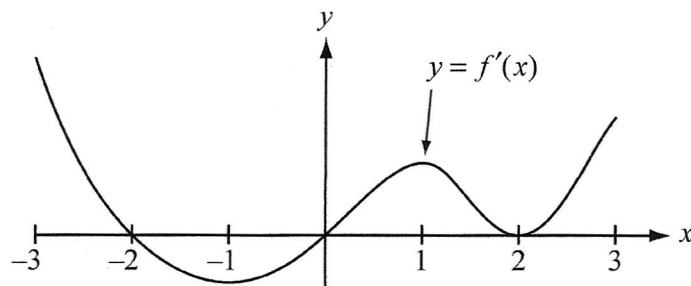
- (a) For what values of  $x$  is the function increasing?
- (b) Find the  $x$ - and  $y$ -coordinates of the relative maximum and minimum points. Justify your answer.
- (c) For what values of  $x$  is the graph of  $f$  concave upward?
- (d) Using the information found in parts (a), (b), and (c), sketch the graph of  $f$  on the axes provided.



Let  $f$  be the function defined by  $f(x) = 12x^{\frac{2}{3}} - 4x$ .

- (a) Find the intervals on which  $f$  is increasing.
- (b) Find the  $x$ - and  $y$ -coordinates of all relative maximum points.
- (c) Find the  $x$ - and  $y$ -coordinates of all relative minimum points.
- (d) Find the intervals on which  $f$  is concave downward.
- (e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of  $f$  on the axes provided.





Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

The figure above shows the graph of  $f'$ , the derivative of a function  $f$ . The domain of the function  $f$  is the set of all  $x$  such that  $-3 \leq x \leq 3$ .

- For what values of  $x$ ,  $-3 < x < 3$ , does  $f$  have a relative maximum? A relative minimum? Justify your answer.
- For what values of  $x$  is the graph of  $f$  concave up? Justify your answer.
- Use the information found in parts (a) and (b) and the fact that  $f(-3) = 0$  to sketch a possible graph of  $f$  on the axes provided below.

