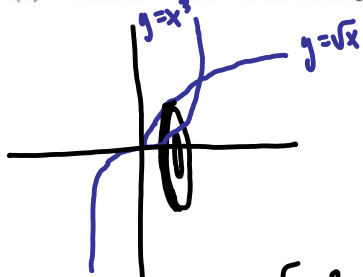
1980 AB1

Let R be the region enclosed by the graphs of $y = x^3$ and $y = \sqrt{x}$.

(a) Find the area of R.

(b) Find the volume of the solid generated by revolving R about the x-axis.



$$A = \int_0^1 x^{1/2} - x^3\,dx = \left[\frac{2}{3}x^{3/2} - \frac{x^4}{4}\right]_0^1 = \frac{2}{3} - \frac{1}{4} = \boxed{\frac{5}{12}}$$

$$V = \pi\int_0^1 x - x^6\,dx = \pi\left[\frac{1}{2}x^2 - \frac{1}{7}x^7\right]_0^1 = \pi\left[\frac{1}{2} - \frac{1}{7}\right] = \boxed{\frac{5\pi}{14}}$$

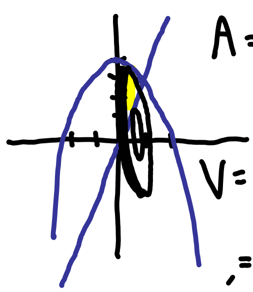
1981 AB2

Let R be the region in the <u>first quadrant</u> enclosed by the graphs of $y = 4 - x^2$, $y = 3x$, and the <u>y-axis</u>.

(a) Find the area of region R.

(b) Find the volume of the solid formed by revolving the region R about the <u>x-axis</u>.

$$4 - x^2 = 3x$$
$$x^2 + 3x - 4 = 0$$
$$(x+4)(x-1) = 0$$



$$A = \int_0^1 4 - x^2 - 3x\,dx = \left[4x - \frac{x^3}{3} - \frac{3x^2}{2}\right]_0^1 = 4 - \frac{1}{3} - \frac{3}{2} = \boxed{\frac{13}{6}}$$

$$V = \pi\int_0^1 (4-x^2)^2 - (3x)^2\,dx$$
$$= \pi\int_0^1 16 - 8x^2 + x^4 - 9x^2\,dx = \pi\int_0^1 x^4 - 17x^2 + 16\,dx$$
$$= \pi\left[\frac{x^5}{5} - \frac{17}{3}x^3 + 16x\right]_0^1 = \pi\left[\frac{1}{5} - \frac{17}{3} + 16\right] = \boxed{\frac{158\pi}{15}}$$

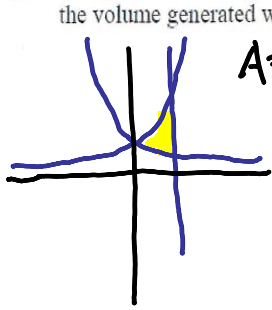
1985 AB3

Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^x$, and $x = \ln 4$.

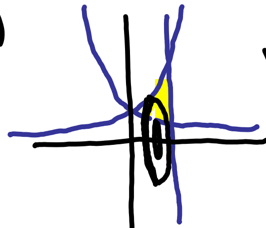
(a) Find the area of R by setting up and evaluating a definite integral.

(b) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the <u>x-axis</u>.

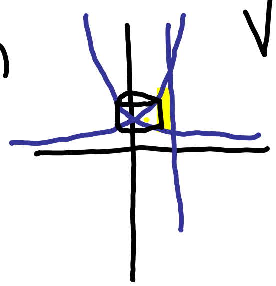
(c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the <u>y-axis</u>.



$$A = \int_0^{\ln 4} e^x - e^{-x}\,dx = \left[e^x + e^{-x}\right]_0^{\ln 4} = e^{\ln 4} + e^{-\ln 4} - 2$$
$$= 4 + \frac{1}{4} - 2 = \boxed{\frac{9}{4}}$$

b) 

$$V=\pi\int_0^{\ln 4} (e^x)^2-(e^{-x})^2\,dx$$

c) 

$$V=2\pi\int_0^{\ln 4} x\,(e^x-e^{-x})\,dx$$

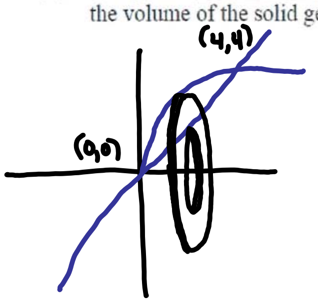
1987 AB3

Let R be the region enclosed by the graphs of $y=(64x)^{\frac{1}{4}}$ and $y=x$.

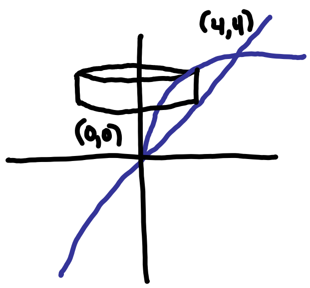
$$(64x)^{1/4}=x$$
$$64x=x^4$$
$$x^4-64x=0$$
$$x(x^3-64)=0$$
$$x(x-4)(x^2+4x+16)=0$$
$$x=0 \quad x=4$$

(a) Find the volume of the solid generated when region R is revolved about the x-axis.

(b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when region R is revolved about the y-axis.



$$V=\pi\int_0^4 (64x)^{1/2}-x^2\,dx$$
$$=\pi\int_0^4 8x^{1/2}-x^2\,dx=\pi\left[\frac{16}{3}x^{3/2}-\frac{x^3}{3}\right]_0^4$$
$$=\pi\left[\frac{128}{3}-\frac{64}{3}\right]=\frac{64\pi}{3}$$



$$V=2\pi\int_0^4 x\left[(64x)^{1/4}-x\right]dx$$

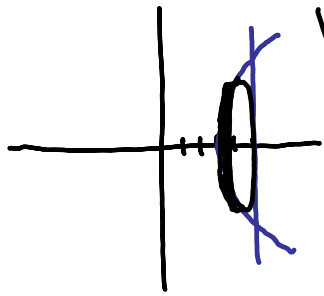
1988 AB3

Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

$y = \sqrt{x^2 - 9}$

(a) Find the volume of the solid generated by revolving R about the <u>x-axis</u>.

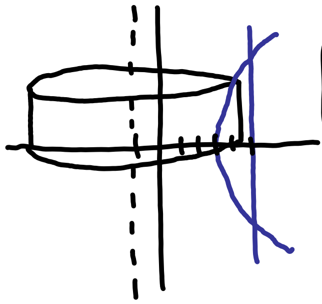
(b) Set up, <u>but do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line $x = -1$.



$$V = \pi\int_3^5 \left(\sqrt{x^2-9}\right)^2 dx = \pi\int_3^5 x^2 - 9\, dx$$

$$= \pi\left[\frac{x^3}{3} - 9x\right]_3^5 = \pi\left[\left(\frac{125}{3} - 45\right) - (9 - 27)\right]$$

$$= \boxed{\frac{44\pi}{3}}$$



$$\boxed{V = 2\pi\int_3^5 (x+1)\sqrt{x^2-9}\, dx}$$