AP Calc BC Problem Set #3

Let p and q be real numbers and let f be the function defined by:

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2, & \text{for } x \le 1 \\ qx + p, & \text{for } x > 1. \end{cases}$$

(a) Find the value of q, in terms of p, for which f is continuous at $x = 1$.

(b) Find the values of p and q for which f is differentiable at $x = 1$.

(c) If p and q have the values determined in part (b), is f'' a continuous function? Justify your answer.

Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$.

(c) Find the x-coordinate of each point at which the line tangent to the graph of f is parallel to the line $y = -2x + 4$.

Let f be the real-valued function defined by $f(x) = \sin^3 x + \sin^3 |x|$.

(a) Find $f'(x)$ for $x > 0$.

(b) Find $f'(x)$ for $x < 0$.

(c) Determine whether $f(x)$ is continuous at $x = 0$. Justify your answer.

(d) Determine whether the derivative of $f(x)$ exists at $x = 0$. Justify your answer.

Let f be the real-valued function defined by $f(x) = \sqrt{1+6x}$.

(a) Give the domain and range of f.

(b) Determine the slope of the line tangent to the graph of f at $x = 4$.

(c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.

(d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

Let f be the function defined by $f(x) = \dfrac{2x-5}{x^2-4}$.

(a) Find the domain of f.

(b) Write an equation for each vertical and each horizontal asymptote for the graph of f.

(c) Find $f'(x)$.

(d) Write an equation for the line tangent to the graph of f at the point $(0, f(0))$.

Let $f(x) = \sqrt{1 - \sin x}$.

(a) What is the domain of f?

(b) Find $f'(x)$.

(c) What is the domain of f'?

(d) Write an equation for the line tangent to the graph of f at $x = 0$.

Let f be the function given by $f(x)=\sqrt{x^4-16x^2}$.

(a) Find the domain of f.

(b) Describe the symmetry, if any, of the graph of f.

(c) Find $f'(x)$.

(d) Find the slope of the line normal to the graph of f at $x=5$.

A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x = \frac{1}{3}t^3 - 3t^2 + 8t$.

(a) Show that at time $t = 0$, the particle is moving to the right.

(b) Find all values of t for which the particle is moving to the left.

(c) What is the position of the particle at time $t = 3$?

(d) When $t = 3$, what is the total distance the particle has traveled?

A particle starts at time $t = 0$ and moves on a number line so that its position at time t is given by $x(t) = (t-2)^3(t-6)$.

(a) When is the particle moving to the right?

(b) When is the particle at rest?

(c) When does the particle change direction?

(d) What is the farthest to the left of the origin that the particle moves?

A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

(a) Find the velocity at time t.

(b) Find the acceleration at time t.

(c) What are all values of t, $0 \le t \le 3$, for which the particle is at rest?

(d) What is the maximum velocity?

Given the curve $x + xy + 2y^2 = 6$.

(a) Find an expression for the slope of the curve at any point (x, y) on the curve.

(b) Write an equation for the line tangent to the curve at the point $(2,1)$.

(c) Find the coordinates of all other points on this curve with slope equal to the slope at $(2,1)$.

Let $y = f(x)$ be the continuous function that satisfies the equation $x^4 - 5x^2y^2 + 4y^4 = 0$ and whose graph contains the points $(2,1)$ and $(-2,-2)$. Let ℓ be the line tangent to the graph of f at $x = 2$.

(a) Find an expression for y'.

(b) Write an equation for line ℓ.

(c) Give the coordinates of a point that is on the graph of f but is not on line ℓ.

(d) Give the coordinates of a point that is on line ℓ but is not on the graph of f.

Consider the curve given by the equation $y^3 + 3x^2y + 13 = 0$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(2, -1)$.

(c) Find the minimum y-coordinate of any point on the curve. Justify your answer.

Given the curve $x^2 - xy + y^2 = 9$.

(a) Write a general expression for the slope of the curve.

(b) Find the coordinates of the points on the curve where the tangents are vertical.

(c) At the point $(0,3)$ find the rate of change in the slope of the curve with respect to x.