Calculus A Basic Differentiation Rules Test Review Name:

Find $f'(x)$ for the following functions.

1. $f(x) = x^3$ $f'(x) = \boxed{3x^2}$

2. $f(x) = \frac{1}{x^5} = x^{-5}$ $f'(x) = -5x^{-6} = \boxed{\frac{-5}{x^6}}$

3. $f(x) = 4x^5 + 3x^4 + 2x^2 + 5$ $f'(x) = \boxed{20x^4 + 12x^3 + 4x}$

4. $f(x) = -2x^2 - 5\cos x$ $f'(x) = -4x - 5(-\sin x)$
$= \boxed{-4x + 5\sin x}$

5. $f(x) = \underbrace{x^3}_{1} \underbrace{\sec x}_{2}$ $f'(x) = \underbrace{x^3}_{1} \underbrace{\sec x \tan x}_{D2} + \underbrace{\sec x}_{2} \underbrace{(3x^2)}_{D1}$
$= \boxed{x^2 \sec x (x\tan x + 3)}$

6. $f(x) = \frac{x+2}{3x-1}$

$$f'(x) = \frac{(3x-1)(1) - (x+2)(3)}{(3x-1)^2}$$

$$= \frac{3x-1-3x-6}{(3x-1)^2}$$

$$= \boxed{\frac{-7}{(3x-1)^2}}$$

Find the slope of the graph at the given value.

7. $f(x) = \frac{-3}{x^2}$ when $x = 2$

$$f(x) = -3x^{-2}$$

$$f'(x) = 6x^{-3} = \boxed{\frac{6}{x^3}}$$

$$f'(2) = \frac{6}{(2)^3} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

8. $f(x) = 3x^3 + 2x^5 - 5$ when $x = 1$

$$f'(x) = 9x^2 + 10x^4$$

$$f'(1) = 9(1)^2 + 10(1)^4 = 9+10 = \boxed{19}$$

9. $f(x) = 2(3x+1)^2$ when $x = 1$

$$f(x) = 2(9x^2+6x+1) = 18x^2+12x+2$$

$$f'(x) = 36x+12$$

$$f'(1) = 36(1)+12 = \boxed{48}$$

$$f'(x) = 4(3x+1)(3)$$

$$f'(1) = 4(3(1)+1)(3)$$

$$= 16 \cdot 3 = \boxed{48}$$

10. $f(x) = \boxed{\boxed{(\boxed{2x+4})^2}\tan x}$ when $x = 0$

$\sec 0 = \frac{1}{\cos 0} = 1$

$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

$f'(x) = (2x+4)^2 \sec^2 x + \tan x \cdot 2(2x+4)(2)$

$f'(0) = 16(1)^2 + 0 = \boxed{16}$

11. $f(x) = x^{\frac{1}{2}} - 3x^{-\frac{3}{2}}$ when $x = 1$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{3}{2}(-3)x^{-5/2}$

$= \frac{1}{2}(1) + \frac{9}{2}(1) = \frac{10}{2} = \boxed{5}$

12. $f(x) = \frac{3x^2 \sin x}{x+2}$ when $x = 0$

$\frac{d}{dx}\left[3x^2 \sin x\right]$

$f'(x) = \frac{(x+2)(3x^2\cos x + 6x\sin x) - 3x^2 \sin x}{(x+2)^2}$

$= 3x^2 \cos x + \sin x \cdot 6x$

$= 3x^2 \cos x + 6x \sin x$

$f'(0) = \frac{2(0) - 0}{4} = \boxed{0}$

Determine the x-value(s) at which the graph of the function has a horizontal tangent.

set $f'(x) = 0$, then solve for x

13. $f(x) = 2x^3 - 3x^2 - 36x$

$= 6x^2 - 6x - 36$

$= 6(x^2 - x - 6)$

$= 6(x-3)(x+2)$

$\boxed{x=3}$ $\boxed{x=2}$

Find the second derivative of the function.

14. $f(x) = 3x^2 + 15x - 3$

$f'(x) = 6x + 15$

$f''(x) = \boxed{6}$

15. $f(x) = \boxed{2x}\boxed{\sin x}$

$f'(x) = 2x\cos x + 2\sin x$

$f''(x) = 2x(-\sin x) + 2\cos x + 2\cos x$

$= \boxed{4\cos x - 2x\sin x}$

Find the second derivative of the function at the given value.

16. $f(x) = \frac{5x^2+2x}{x+1}$ when $x = 2$

$$f'(x) = \frac{(x+1)(10x+2) - (5x^2+2x)}{(x+1)^2} = \frac{10x^2+12x+2-5x^2-2x}{(x+1)^2}$$

$$= \frac{5x^2+10x+2}{(x+1)^2}$$

$$f''(x) = \frac{(x+1)^2(10x+10) - (5x^2+10x+2)\,2(x+1)}{(x+1)^4}$$

$$f''(2) = \frac{9(30) - 42\cdot 6}{81} = \frac{270-252}{81} = \frac{18}{81} = \boxed{\frac{2}{9}}$$

Find the 1st, 2nd, 3rd, and 4th derivative of the function. Use $f'(x)$ for the first derivative, $f''(x)$ for the second derivative, $f'''(x)$ for the third derivative, and $f^{(4)}(x)$ for the fourth derivative.

17. $f(x) = 3x^{\frac{7}{3}}$

$$f'(x) = \frac{7}{3}\cdot 3x^{4/3} = 7x^{4/3}$$

$$f''(x) = \frac{4}{3}\cdot 7x^{4/3} = \frac{28}{3}x^{4/3}$$

$$f'''(x) = \frac{4}{3}\,\frac{28}{3}\,x^{1/3} = \frac{112}{9}x^{1/3}$$

$$f^{(4)}(x) = \frac{1}{3}\,\frac{112}{9}\,x^{-2/3} = \boxed{\frac{112}{27}x^{-2/3}}$$

Find the equation of the tangent line to graph of $f(x)$ at the given value.

18. $f(x) = 2x^3 + 3x^2 + 1$ at $x = 3$

$y - y_1 = m(x - x_1)$

$f'(x) = 6x^2 + 6x$

$f'(3) = 6(3)^2 + 6(3) = 54 + 18 = 72$

$f(3) = 2(3)^3 + 3(3)^2 + 1$

$= 54 + 27 + 1$

$= 82$

$m = f'(3) = 72$

$x_1 = 3$

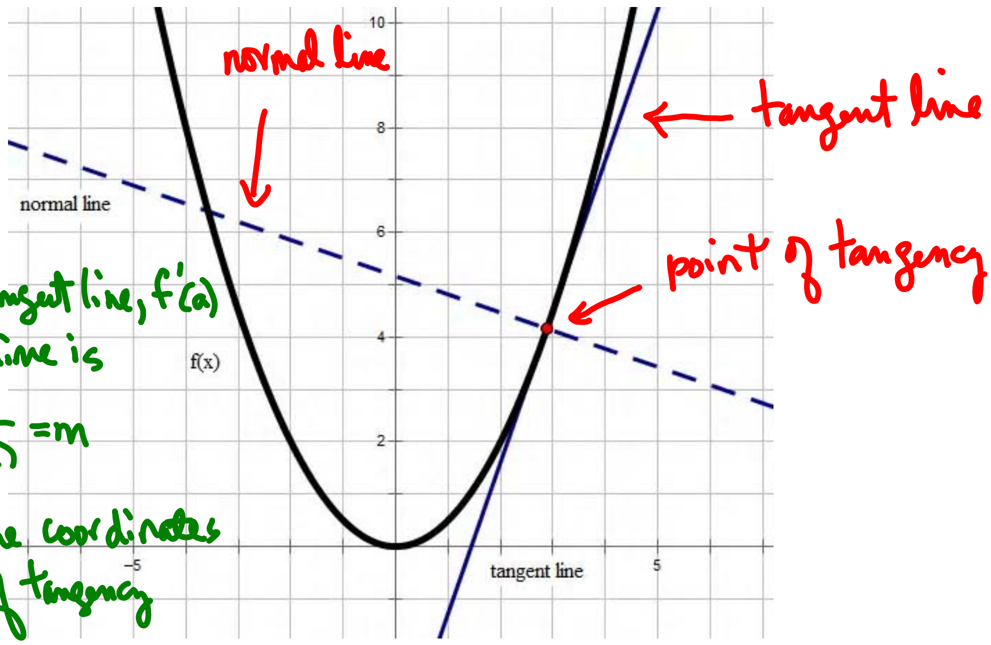
$y_1 = f(3) = 82$

$$\boxed{y - 82 = 72(x - 3)}$$

Find the equation of the to the graph of $f(x)$ at the given value. A normal line is perpendicular to the tangent line and passes through the point of tangency. Its slope is the opposite sign and reciprocal of the slope of the tangent line. You can use the point of tangency and slope of the normal line to find the *equation* of the normal line. See the example below:

$y - y_1 = m(x - x_1)$

1. find slope of tangent line, $f'(a)$
2. Slope of normal line is $-\frac{1}{f'(a)} = m$
3. (x_1, y_1) are the coordinates of the point of tangency



19. $f(x) = \frac{x+3}{2x-5}$ at $x = 2$

$f'(x) = \frac{(2x-5)(1) - (x+3)(2)}{(2x-5)^2}$

$f'(2) = \frac{-1 - 10}{1} = -11$

$m_{tan} = -11 \quad m_{normal} \ \frac{1}{11}$

$f(2) = \frac{5}{-1} = -5 \quad (-2, -5)$

$y - y_1 = m(x - x_1)$

$\boxed{y + 5 = \frac{1}{11}(x + 2)}$