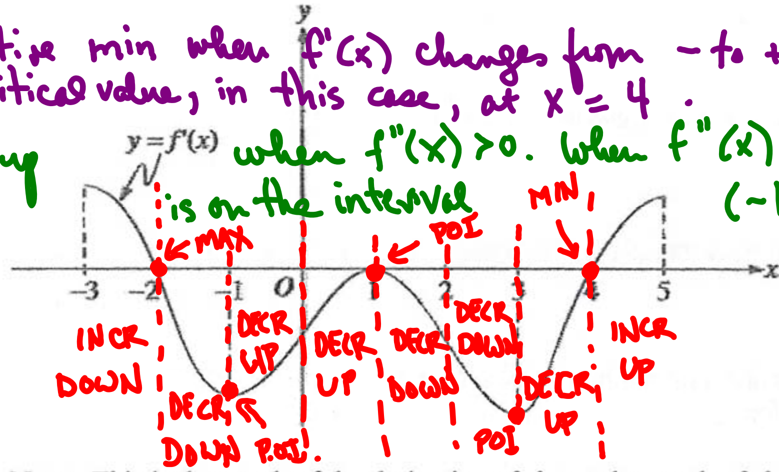
a. $f(x)$ has a relative max when $f'(x)$ changes from + to −. This always occurs at a critical value, in this case, at $x = -2$.

1996 AB1

b. $f(x)$ has a relative min when $f'(x)$ changes from − to +. This always occurs at a critical value, in this case, at $x = 4$.

c. $f(x)$ is concave up when $f''(x) > 0$. When $f''(x) > 0$, $f'(x)$ increases. In this case, it is on the interval $(-1,1) \cup (3,5)$.

DECREASING AND CONCAVE DOWN



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$. ← note: open interval

(a) For what values of x does f have a relative maximum? Why?

(b) For what values of x does f have a relative minimum? Why?

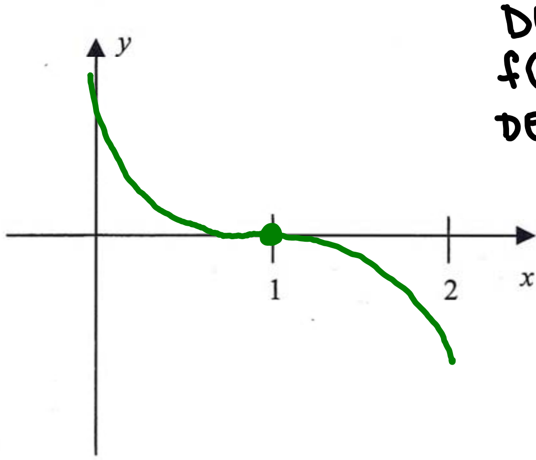
(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

(d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

DECREASING & CONCAVE UP $0 < x < 1$

$f(1) = 0$ and $f'(1) = 0$

DECREASING & CONCAVE DOWN $1 < x < 2$



$f'(x) = 12x^3+3x^2-42x = 3x(4x^2+x-14) = 3x(4x^2+8x-7x-14)$

$= 3x(4x-7)(x+2)$

$= 3x[4x(x+2)-7(x+2)]$

$f'(2) = 6(1)(4) = 24$

-56
8 -7
1

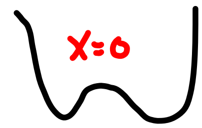
1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

$= x^2(3x^2+x-21)$

(a) Write an equation of the line tangent to the graph of f at the point $(2, -28)$.

$y - y_1 = m(x - x_1)$ $y+28 = 24(x-2)$



x=0

x=-2 x=7/4

(b) Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

$f'(x)=0$ when $x=0$ $x=7/4$ $x=-2$

Since this is not a closed interval, endpoints don't need to be checked.

ABS MIN $\rightarrow f(-2) = -44$

$f(0) = 0$

$f(7/4) = -30.816$

(c) Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

$f''(x) = 36x^2 + 6x - 42 = 0$ when $6(6x^2+x-7)=0$

$x=1$, $x=-7/6$ ← points of inflection!!

-42
7 -6
1

$6[6x^2+7x-6x-7]$

$6[x(6x+7)-(6x+7)]$

$\rightarrow 6(x-1)(6x+7)$

$f''(x)$: + | − | +

-2 -7/6 0 1 2

1993 AB4/BC3

Extreme Value Theorem (EVT) - a continuous function on a closed interval always has an absolute min and max

Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$. ← closed interval

(a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.

(b) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.

a) abs. min and max occur at critical numbers or end points when EVT holds. Evaluate $f(x)$ at those points. Lowest is abs. min. Highest is abs. max.

$$f'(x) = \frac{1}{2+\sin x}(\cos x) = \frac{\cos x}{2+\sin x} = 0 \text{ when } \cos x = 0 \text{ @ } x = \cancel{\tfrac{\pi}{2}}, \tfrac{3\pi}{2}$$ is the only critical value.

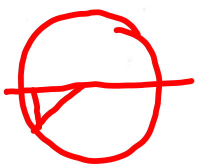
$f(\pi) = \ln 2$ abs. max

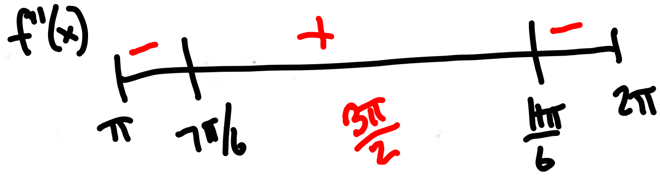
$f\left(\frac{3\pi}{2}\right) = 0$ abs. min

$f(2\pi) = \ln 2$

b) $$f''(x) = \frac{(2+\sin x)(-\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2} = \frac{-2\sin x - 1(\sin^2 x + \cos^2 x)}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2} = 0 \text{ when } 2\sin x = -1, \ \sin x = -1/2 \text{ @ } x = \frac{7\pi}{6}, \frac{11\pi}{6}$$





Points of inflection are $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

1993 AB1

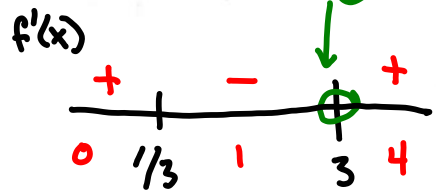
Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

(a) On what intervals is f increasing?

(b) On what intervals is the graph of f concave downward?

(c) Find the value of k for which f has 11 as its relative minimum.

rel. min. @ $x=3$



a) $f(x)$ increases when $f'(x) > 0$

$f'(x) = 3x^2 - 10x + 3 = 0$ when $3x^2 - 9x - 1x + 3 = 0$

9, −9, −1, −10

$3x(x-3) - (x-3) = 0$

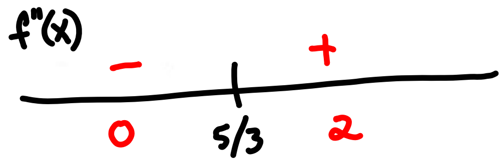
$(3x-1)(x-3) = 0$

$x = 1/3 \quad x = 3$

$f(x)$ increases on $(-\infty, \frac{1}{3}) \cup (3, \infty)$

b) $f(x)$ is concave down when $f''(x) < 0$

$f''(x) = 6x - 10 = 0$ when $x = \frac{10}{6} = \frac{5}{3}$



$f(x)$ is concave down on $(-\infty, 5/3)$

c) $f(x)$ has rel. min when $f'(x)$ changes from − to +.

$f(3) = (3)^3 - 5(3)^2 + 3(3) + k = 11$

$27 - 45 + 9 + k = 11$

$-9 + k = 11$

$\therefore k = 20$

1992 AB1

$\sqrt{2} \approx 1.4$
$\sqrt{3} \approx 1.7$

Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

(a) On what intervals is f increasing?

(b) On what intervals is the graph of f concave upward?

(c) Write the equation of each horizontal tangent line to the graph of f.

a) $f(x)$ increases when $f'(x) > 0$

$f'(x) = 15x^4 - 15x^2 = 0$ when $15x^2(x^2-1) = 0$ or $15x^2(x+1)(x-1) = 0$

$x = -1 \quad x = 0 \quad x = 1$

$f'(x)$: $+$ | $-$ | $-$ | $+$

$-2 \quad -1 \quad -1/2 \quad 0 \quad 1/2 \quad 1 \quad 2$

$f(x)$ increases on $(-\infty, -1) \cup (1, \infty)$

b) $f(x)$ is concave up when $f''(x) > 0$

$f''(x) = 60x^3 - 30x = 0$ when $30x(2x^2-1) = 0$ when $x = 0$ and $2x^2 - 1 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$f''(x)$: $-$ | $+$ | $-$ | $+$

$-1 \quad -\sqrt{2}/2 \quad -1/2 \quad 0 \quad 1/2 \quad \sqrt{2}/2 \quad 1$

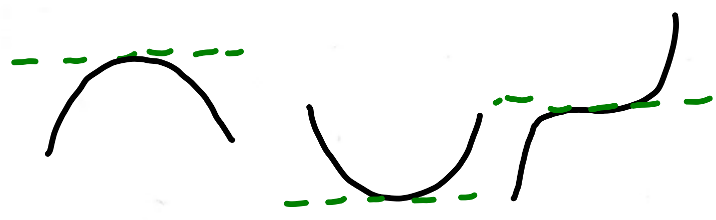
$f(x)$ is concave up on $\left(-\frac{\sqrt{2}}{2}, 0\right) \cup \left(\sqrt{2}/2, \infty\right)$

c) $f(x) = 3x^5 - 5x^3 + 2$

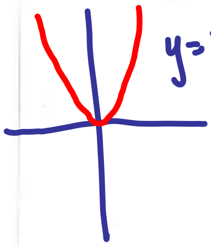
$f(-1) = -3 + 5 + 2 = 4$ $\quad y = 4$

$f(0) = 2$ $\quad y = 2$

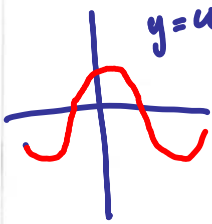
$f(1) = 0$ $\quad y = 0$



$y=x^2$



$y=\cos x$



1991 AB5

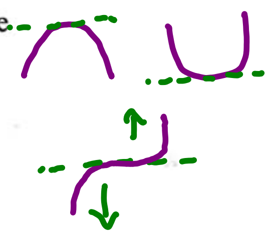
$(0,1)$ is a critical value either a sharp turn or vertical tangent

Let f be a function that is even and continuous on the closed interval $[-3,3]$. The function f and its derivatives have the properties indicated in the table below.

$(1,0)$ is a critical value. It's a horizontal tangent

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

+ POI − −



(a) Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each x-coordinate you give, state whether f attains an absolute maximum or an absolute minimum.

Abs. max occurs at critical values or end points

$f(-3)$

$f(0)=1 \leftarrow$ abs. max

$f(1)=0$

$f(2)=-1 \leftarrow$ abs. min $\rightarrow f(-2)=-1$

$f(3)$

(b) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer. $x=1$ and $x=-1$ are the only points on the graph where $f''(x)$ changes signs

(c) In the xy-plane provided below, sketch the graph of a function with all the given characteristics of f.

$(2,-1)$ is a critical value either a sharp turn or vertical tangent

abs. max