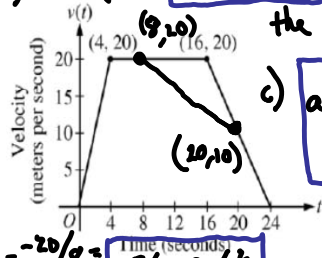
a) $\int_0^{24} v(t)\,dt = \frac{20}{2}(24+12) = 10(36) = \boxed{360 \text{ meters.}}$ This is $\boxed{\text{distance traveled}}$ in the first 24 seconds.

b) $v'(t)$ is the slope of $v(t)$ at time t.

$\boxed{v'(4) \text{ DNE}}$ because $\lim_{t\to 4^-} v'(t) \neq \lim_{t\to 4^+} v'(t)$

$v'(20) = (0-20)/(24-16) = -20/8 = \boxed{-5/2 \text{ m/s}^2}$



c) $$\boxed{a(t) = \begin{cases} 5, & 0<t<4 \\ 0, & 4<t<16 \\ -5/2 & 16<t<24 \end{cases}}$$

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

d) $(10-20)/(20-8) = -10/12 = \boxed{-5/6}$

$\boxed{\text{MVT does not apply because } v(t) \text{ is not differentiable on } (8,24)}$

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

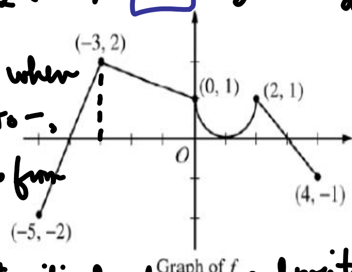
(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

a) $g(0) = \int_{-3}^{0} f(x)\,dx = \frac{3}{2}(2+1) = \boxed{9/2}$ $\quad g'(x) = \frac{d}{dx}\left[\int_{-3}^{x} f(t)\,dt\right] = f(x)$

and $g'(0) = f(0) = \boxed{1}$

b) $g(x)$ has rel. max when $g'(x)$ changes from + to −, i.e. when $f(x)$ changes from + to −, $\boxed{x=3.}$



x	$g(x)$
−5	0
−4	−1
1	$11/2 - \pi/4$, $> g(-4)$
3	$> g(1) \Rightarrow > g(-4)$
4	$> g(1) \Rightarrow > g(-4)$

c) Abs. min. occurs at critical values or endpoints. i.e. @ $\boxed{x=-4}$

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

$g(-5) = \int_{-3}^{-5} f(x)\,dx = -\int_{-5}^{-3} f(x)\,dx = 0$ $\quad g(1) = \frac{9}{2} + 1 - \frac{\pi}{4} = \frac{11}{2} - \frac{\pi}{4}$

$g(-4) = \int_{-3}^{-4} f(x)\,dx = -\int_{-4}^{-3} f(x)\,dx = -1$

$g(3) > g(1)$

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

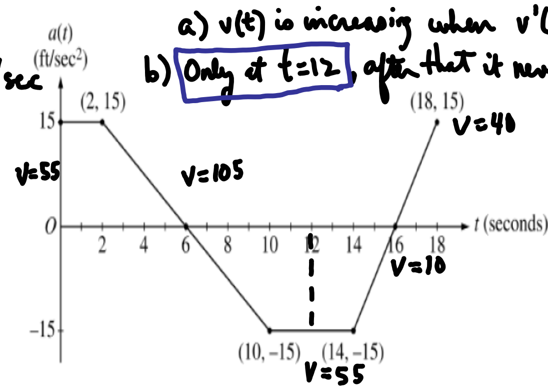
(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

d) $g(x)$ has pt. of inflection when $g''(x)$ changes sign. $g''(x) = f'(x)$ and $f'(x)$ changes sign $\boxed{@\, x=-3,\ x=1,\ x=2}$

$v(0) = 55$ ft/sec

a) $v(t)$ is increasing when $v'(t) > 0$ since $a(2) > 0$, $\boxed{v(t) \text{ is increasing}}$

b) $\boxed{\text{Only at } t=12}$, after that it never reaches 0 again.



c) max velocity occurs at critical values ($a(t)=0$) or at endpoints

t	$v(t)$	
0	55	
6	105	← ABS. MAX
16	10	← ABS. MIN
18	40	

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec², is the piecewise linear function defined by the graph above.

d) $\boxed{\text{Since Abs. Min} = 10, \text{ velocity never reaches } 0.}$

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

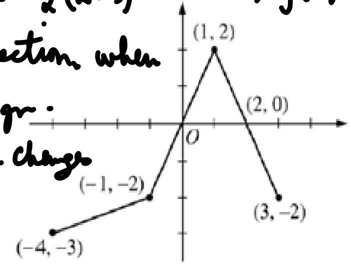
(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

a) $g(-1) = \int_{-4}^{-1} f(x)\,dx = -\frac{3}{2}(2+3) = -15/2$, $g'(-1) = f(-1) = -2$ $g''(-1) = f'(-1)$ DNE

b) $g(x)$ has pt. of inflection when $g''(x)$ changes sign. $g''(x) = f'(x)$ which changes sign @ $x=1$

c) $x=1, x=-1$

d) h is decreasing when $h'(x)<0$ since $h(x) = \int_x^3 f(t)\,dt = -\int_3^x f(t)\,dt$ so $h'(t) = -f(x)$ and $-f(x)<0$ when $f(x)>0$, i.e. on $(0,2)$



Graph of f

The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$. $x=1$ $x=-1$

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

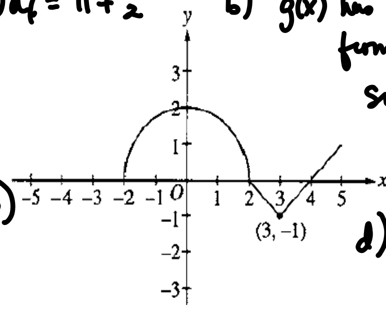
a) $g(3) = \int_0^3 f(x)\,dx = \pi + \frac{1}{2}$

b) $g(x)$ has a rel. max when $g'(x)$ changes from + to −. Since $g'(x) = f(x)$, $f(x)$ changes + to − at $x=2$

c) $(3, \pi + \frac{1}{2})$

$m = g'(3) = -1$

$y - \pi + \frac{1}{2} = -1(x-3)$



d) pt. of inflection when $g''(x)$ changes sign $g''(x) = f'(x)$.
$f'(x)$ changes sign at $x=0$, $x=3$

The graph of the function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(3)$.

(b) Find all the values of x on the open interval $(-2,5)$ at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2,5)$. Justify your answer.