Here are some examples that reinforce things that we have learned up to this point.

The exercises in this section are designed to reinforce your understanding of the definite integral from the algebraic and geometric points of view. For this reason, you should not use the numerical integration capability of your calculator (NINT) except perhaps to support an answer.

1. Suppose that f and g are continuous functions and that

$$\int_1^2 f(x)\,dx = -4, \quad \int_1^5 f(x)\,dx = 6, \quad \int_1^5 g(x)\,dx = 8.$$

Use the rules in Table 5.3 to find each integral.

(a) $\int_2^2 g(x)\,dx$ 0 **(b)** $\int_5^1 g(x)\,dx$ -8

(c) $\int_1^2 3f(x)\,dx$ -12 **(d)** $\int_2^5 f(x)\,dx$ 10

(e) $\int_1^5 [f(x) - g(x)]\,dx$ -2 **(f)** $\int_1^5 [4f(x) - g(x)]\,dx$ 16

2. Suppose that f and h are continuous functions and that

$$\int_1^9 f(x)\,dx = -1, \quad \int_7^9 f(x)\,dx = 5, \quad \int_7^9 h(x)\,dx = 4.$$

Use the rules in Table 5.3 to find each integral.

(a) $\int_1^9 -2f(x)\,dx$ 2 **(b)** $\int_7^9 [f(x) + h(x)]\,dx$ 9

(c) $\int_7^9 [2f(x) - 3h(x)]\,dx$ -2 **(d)** $\int_9^1 f(x)\,dx$ 1

(e) $\int_1^7 f(x)\,dx$ -6 **(f)** $\int_9^7 [h(x) - f(x)]\,dx$ 1

3. Suppose that $\int_1^2 f(x)\,dx = 5$. Find each integral.

(a) $\int_1^2 f(u)\,du$ 5 **(b)** $\int_1^2 \sqrt{3}\,f(z)\,dz$ $5\sqrt{3}$

(c) $\int_2^1 f(t)\,dt$ -5 **(d)** $\int_1^2 [-f(x)]\,dx$ -5

4. Suppose that $\int_{-3}^0 g(t)\,dt = \sqrt{2}$. Find each integral.

(a) $\int_0^{-3} g(t)\,dt$ $-\sqrt{2}$ **(b)** $\int_{-3}^0 g(u)\,du$ $\sqrt{2}$

(c) $\int_{-3}^0 [-g(x)]\,dx$ $-\sqrt{2}$ **(d)** $\int_{-3}^0 \frac{g(r)}{\sqrt{2}}\,dr$ 1

In Exercises 19–30, evaluate the integral using antiderivatives, as in Example 4.

19. $\int_\pi^{2\pi} \sin x\,dx$ $-\cos(2\pi) + \cos\pi = -2$ **20.** $\int_0^{\pi/2} \cos x\,dx$ $\sin(\pi/2) - \sin 0 = 1$

21. $\int_0^1 e^x\,dx$ $e^1 - e^0 = e - 1$ **22.** $\int_0^{\pi/4} \sec^2 x\,dx$ $\tan(\pi/4) - \tan 0 = 1$

23. $\int_1^4 2x\,dx$ $4^2 - 1^2 = 15$ **24.** $\int_{-1}^2 3x^2\,dx$ $2^3 - (-1)^3 = 9$

25. $\int_{-2}^6 5\,dx$ $5(6) - 5(-2) = 40$ **26.** $\int_3^7 8\,dx$ $8(7) - 8(3) = 32$

27. $\int_{-1}^1 \frac{1}{1+x^2}\,dx$ $\tan^{-1}(1) - \tan^{-1}(-1) = \pi/2$ **28.** $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$ $\sin^{-1}(1/2) - \sin^{-1}(0) = \pi/6$

29. $\int_1^e \frac{1}{x}dx$ $\ln e - \ln 1 = 1$ **30.** $\int_1^4 -x^{-2}\,dx$ $4^{-1} - 1^{-1} = -3/4$

In Exercises 1–20, find dy/dx.

1. $y = \int_0^x (\sin^2 t)\, dt$ $\quad \sin^2 x$

2. $y = \int_2^x (3t + \cos t^2)\, dt$ $\quad 3x + \cos x^2$

3. $y = \int_0^x (t^3 - t)^5\, dt$ $\quad (x^3 - x)^5$

4. $y = \int_{-2}^x \sqrt{1 + e^{5t}}\, dt$ $\quad \sqrt{1 + e^{5x}}$

5. $y = \int_2^x (\tan^3 u) du$ $\quad \tan^3 x$

6. $y = \int_4^x e^u \sec u\, du$ $\quad e^x \sec x$

7. $y = \int_7^x \frac{1 + t}{1 + t^2}\, dt$ $\quad \frac{1 + x}{1 + x^2}$

8. $y = \int_{-\pi}^x \frac{2 - \sin t}{3 + \cos t}\, dt$ $\quad \frac{2 - \sin x}{3 + \cos x}$

9. $y = \int_0^{x^2} e^{t^2}\, dt$ $\quad 2xe^{x^4}$

10. $y = \int_6^{x^2} \cot 3t\, dt$ $\quad 2x \cot 3x^2$

11. $y = \int_2^{5x} \frac{\sqrt{1 + u^2}}{u}\, du$ $\quad \frac{\sqrt{1 + 25x^2}}{x}$

12. $y = \int_\pi^{\pi - x} \frac{1 + \sin^2 u}{1 + \cos^2 u}\, du$

13. $y = \int_x^6 \ln (1 + t^2)\, dt$ $\quad -\ln(1 + x^2)$

14. $y = \int_x^7 \sqrt{2t^4 + t + 1}\, dt$ $\quad -\sqrt{2x^4 + x + 1}$

12. $-\frac{1 + \sin^2(\pi - x)}{1 + \cos^2(\pi - x)}$

15. $y = \int_{x^3}^5 \frac{\cos t}{t^2 + 2}\, dt$ $\quad -\frac{3x^2 \cos x^3}{x^6 + 2}$

16. $y = \int_{5x^2}^{25} \frac{t^2 - 2t + 9}{t^3 + 6}\, dt$ $\quad \frac{250x^5 - 100x^3 + 90}{125x^6 + 6}$

17. $y = \int_{\sqrt{x}}^0 \sin(r^2)\, dr$ $\quad -\frac{\sin x}{2\sqrt{x}}$

18. $y = \int_{3x^2}^{10} \ln (2 + p^2)\, dp$ $\quad -6x \ln(2 + 9x^4)$

19. $\int_{x^2}^{x^3} \cos(2t)\, dt$ $\quad 3x^2 \cos(2x^3) - 2x \cos(2x^2)$

20. $y = \int_{\sin x}^{\cos x} t^2\, dt$ $\quad -\sin x \cos^2 x - \cos x \sin^2 x$

In Exercises 21–26, construct a function of the form $y = \int_a^x f(t)\, dt + C$ that satisfies the given conditions.

21. $\frac{dy}{dx} = \sin^3 x$, and $y = 0$ when $x = 5$. $\quad y = \int_5^x \sin^3 t\, dt$

22. $\frac{dy}{dx} = e^x \tan x$, and $y = 0$ when $x = 8$. $\quad y = \int_8^x e^t \tan t\, dt$

23. $\frac{dy}{dx} = \ln(\sin x + 5)$, and $y = 3$ when $x = 2$. $\quad y = \int_2^x \ln (\sin t + 5)\, dt + 3$

24. $\frac{dy}{dx} = \sqrt{3 - \cos x}$, and $y = 4$ when $x = -3$. $\quad y = \int_{-3}^x \sqrt{3 - \cos t}\, dt + 4$

In Exercises 27–40, evaluate each integral using Part 2 of the Fundamental Theorem. Support your answer with NINT if you are unsure.

27. $\int_{1/2}^3 \left(2 - \frac{1}{x}\right) dx$ $\quad 5 - \ln 6 \approx 3.208$

28. $\int_2^{-1} 3^x\, dx$ $\quad -\frac{26}{3 \ln 3} \approx -7.889$

29. $\int_0^1 (x^2 + \sqrt{x})\, dx$ $\quad 1$

30. $\int_0^5 x^{3/2}\, dx$ $\quad 10\sqrt{5} \approx 22.361$

31. $\int_1^{32} x^{-6/5}\, dx$ $\quad \frac{5}{2}$

32. $\int_{-2}^{-1} \frac{2}{x^2}\, dx$ $\quad 1$

33. $\int_0^\pi \sin x\, dx$ $\quad 2$

34. $\int_0^\pi (1 + \cos x)\, dx$ $\quad \pi$

35. $\int_0^{\pi/3} 2 \sec^2\theta\, d\theta$ $\quad 2\sqrt{3}$

36. $\int_{\pi/6}^{5\pi/6} \csc^2\theta\, d\theta$ $\quad 2\sqrt{3}$

37. $\int_{\pi/4}^{3\pi/4} \csc x \cot x\, dx$ $\quad 0$

38. $\int_0^{\pi/3} 4 \sec x \tan x\, dx$ $\quad 4$

39. $\int_{-1}^1 (r + 1)^2\, dr$ $\quad \frac{8}{3}$

40. $\int_0^4 \frac{1 - \sqrt{u}}{\sqrt{u}}\, du$ $\quad 0$