Derivatives by the Limit Process III, Larson 7.0 Section 2.1 #61-88

In Exercises 61–70, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).

61. $f(x) = x^2 - 1, \quad c = 2$

62. $g(x) = x(x - 1), \quad c = 1$

63. $f(x) = x^3 + 2x^2 + 1, \quad c = -2$

64. $f(x) = x^3 + 2x, \quad c = 1$

65. $g(x) = \sqrt{|x|}, \quad c = 0$

66. $f(x) = 1/x, \quad c = 3$

67. $f(x) = (x - 6)^{2/3}, \quad c = 6$

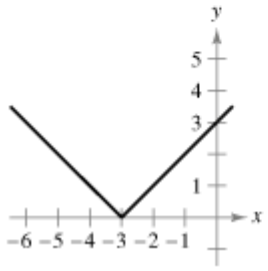
68. $g(x) = (x + 3)^{1/3}, \quad c = -3$

69. $h(x) = |x + 5|, \quad c = -5$

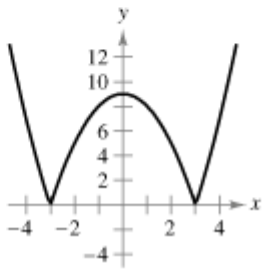
70. $f(x) = |x - 4|, \quad c = 4$

In Exercises 71–80, describe the x-values at which f is differentiable.

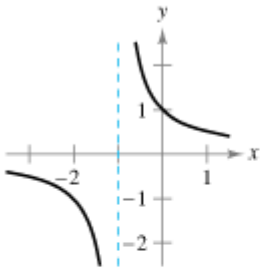
71. $f(x) = |x + 3|$



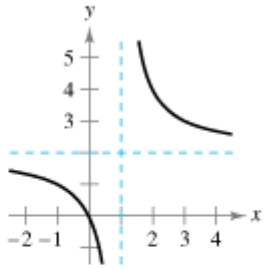
72. $f(x) = |x^2 - 9|$



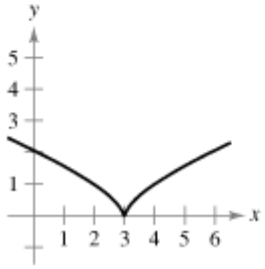
73. $f(x) = \dfrac{1}{x + 1}$



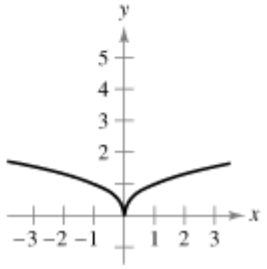
74. $f(x) = \dfrac{2x}{x - 1}$



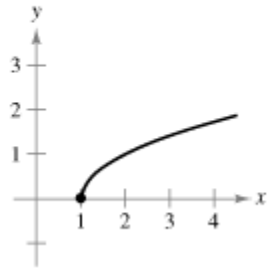
75. $f(x) = (x - 3)^{2/3}$



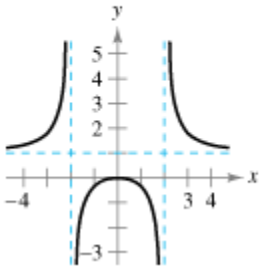
76. $f(x) = x^{2/5}$



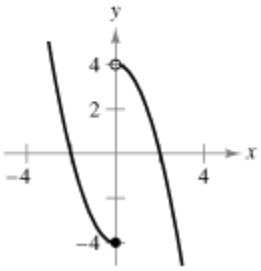
77. $f(x) = \sqrt{x - 1}$



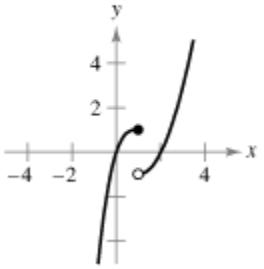
78. $f(x) = \dfrac{x^2}{x^2 - 4}$



79. $f(x) = \begin{cases} 4 - x^2, & x > 0 \\ x^2 - 4, & x \le 0 \end{cases}$



80. $f(x) = \begin{cases} x^2 - 2x, & x > 1 \\ x^3 - 3x^2 + 3x, & x \le 1 \end{cases}$



In Exercises 81–84, find the derivatives from the left and from the right at $x = 1$ (if they exist). Is the function differentiable at $x = 1$?

81. $f(x) = |x - 1|$

82. $f(x) = \sqrt{1 - x^2}$

83. $f(x) = \begin{cases} (x-1)^3, & x \leq 1 \\ (x-1)^2, & x > 1 \end{cases}$

84. $f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$

In Exercises 85 and 86, determine whether the function is differentiable at $x = 2$.

85. $f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 3, & x > 2 \end{cases}$

86. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$

87. ***Graphical Reasoning*** A line with slope m passes through the point $(0, 4)$ and has the equation $y = mx + 4$.

(a) Write the distance d between the line and the point $(3, 1)$ as a function of m.

(b) Use a graphing utility to graph the function d in part (a). Based on the graph, is the function differentiable at every value of m? If not, where is it not differentiable?

88. ***Conjecture*** Consider the functions $f(x) = x^2$ and $g(x) = x^3$.

(a) Graph f and f' on the same set of axes.

(b) Graph g and g' on the same set of axes.

(c) Identify any pattern between the functions f and g and their respective derivatives. Use the pattern to make a conjecture about $h'(x)$ if $h(x) = x^n$, where n is an integer and $n \geq 2$.

(d) Find $f'(x)$ if $f(x) = x^4$. Compare the result with the conjecture in part (c). Is this a proof of your conjecture? Explain.