# Set 3: Multiple-Choice Questions on Differentiation

In each of Questions 1–5 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

**1.** $y = (4x + 1)(1 - x)^3$

**(A)** $-12(1 - x)^2$ **(B)** $(1 - x)^2(1 + 8x)$ **(C)** $(1 - x)^2(1 - 16x)$

**(D)** $3(1 - x)^2(4x + 1)$ **(E)** $(1 - x)^2(16x + 7)$

**2.** $y = \frac{2-x}{3x+1}$

**(A)** $-\frac{7}{(3x+1)^2}$ **(B)** $\frac{6x-5}{(3x+1)^2}$ **(C)** $-\frac{9}{(3x+1)^2}$

**(D)** $\frac{7}{(3x+1)^2}$ **(E)** $\frac{7-6x}{(3x+1)^2}$

**3.** $y = \sqrt{3-2x}$

**(A)** $\frac{1}{2\sqrt{3-2x}}$ **(B)** $-\frac{1}{\sqrt{3-2x}}$ **(C)** $-\frac{(3-2x)^{3/2}}{3}$

**(D)** $-\frac{1}{3-2x}$ **(E)** $\frac{2}{3}(3-2x)^{3/2}$

**4.** $y = \frac{2}{(5x+1)^3}$

**(A)** $-\frac{30}{(5x+1)^2}$ **(B)** $-30(5x+1)^{-4}$ **(C)** $\frac{-6}{(5x+1)^4}$

**(D)** $-\frac{10}{3}(5x+1)^{-4/3}$ **(E)** $\frac{30}{(5x+1)^4}$

**5.** $y = 3x^{2/3} - 4x^{1/2} - 2$

**(A)** $2x^{1/3} - 2x^{-1/2}$ **(B)** $3x^{-1/3} - 2x^{-1/2}$ **(C)** $\frac{9}{5}x^{5/3} - 8x^{3/2}$

**(D)** $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ **(E)** $2x^{-1/3} - 2x^{-1/2}$

In Questions 6–13, differentiable functions $f$ and $g$ have the values shown in the table

| $x$ | $f$ | $f'$ | $g$ | $g'$ |
|---|---|---|---|---|
| 0 | 2 | 1 | 5 | –4 |
| 1 | 3 | 2 | 3 | –3 |
| 2 | 5 | 3 | 1 | –2 |
| 3 | 10 | 4 | 0 | –1 |

6. If $A = f + 2g$, then $A'(3) =$

(A) –2 (B) 2 (C) 7 (D) 8 (E) 10

7. If $B = f \cdot g$, then $B'(2) =$

(A) –20 (B) –7 (C) –6 (D) –1 (E) 13

8. If $D = \frac{1}{g}$, then $D'(1) =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

9. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) 2 (D) $\frac{2}{\sqrt{10}}$ (E) $4\sqrt{10}$

10. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

(A) $\frac{-13}{25}$ (B) $-\frac{1}{4}$ (C) $\frac{13}{25}$ (D) $\frac{13}{16}$ (E) $\frac{22}{25}$

11. If $M(x) = f(g(x))$, then $M'(1) =$

(A) –12 (B) –6 (C) 4 (D) 6 (E) 12

12. If $P(x) = f(x^3)$, then $P'(1) =$

(A) 2 (B) 6 (C) 8 (D) 12 (E) 54

13. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) –2 (B) $-\frac{1}{25}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 2

In Questions 14–21 find $y'$.

**14.** $y = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

**(A)** $x + \dfrac{1}{x\sqrt{x}}$ **(B)** $x^{-1/2} + x^{-3/2}$ **(C)** $\dfrac{4x-1}{4x\sqrt{x}}$

**(D)** $\dfrac{1}{\sqrt{x}} + \dfrac{1}{4x\sqrt{x}}$ **(E)** $\dfrac{4}{\sqrt{x}} + \dfrac{1}{x\sqrt{x}}$

**15.** $y = \sqrt{x^2 + 2x - 1}$

**(A)** $\dfrac{x+1}{y}$ **(B)** $4y(x+1)$ **(C)** $\dfrac{1}{2\sqrt{x^2+2x-1}}$

**(D)** $-\dfrac{x+1}{(x^2+2x-1)^{3/2}}$ **(E)** none of these

**16.** $y = \dfrac{x}{\sqrt{1-x^2}}$

**(A)** $\dfrac{1-2x^2}{(1-x^2)^{3/2}}$ **(B)** $\dfrac{1}{1-x^2}$ **(C)** $\dfrac{1}{\sqrt{1-x^2}}$

**(D)** $\dfrac{1-2x^2}{(1-x^2)^{1/2}}$ **(E)** none of these

**17.** $y = \ln \dfrac{e^x}{e^x - 1}$

**(A)** $x - \dfrac{e^x}{e^x-1}$ **(B)** $\dfrac{1}{e^x-1}$ **(C)** $-\dfrac{1}{e^x-1}$

**(D)** 0 **(E)** $\dfrac{e^x-2}{e^x-1}$

**18.** $y = \tan^{-1} \dfrac{x}{2}$

**(A)** $\dfrac{4}{4+x^2}$ **(B)** $\dfrac{1}{2\sqrt{4-x^2}}$ **(C)** $\dfrac{2}{\sqrt{4-x^2}}$

**(D)** $\dfrac{1}{2+x^2}$ **(E)** $\dfrac{2}{x^2+4}$

**19.** $y = \ln(\sec x + \tan x)$

**(A)** $\sec x$ **(B)** $\dfrac{1}{\sec x}$ **(C)** $\tan x + \dfrac{\sec^2 x}{\tan x}$

**(D)** $\dfrac{1}{\sec x + \tan x}$ **(E)** $-\dfrac{1}{\sec x + \tan x}$

**20.** $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

**(A)** 0 **(B)** 1 **(C)** $\dfrac{2}{(e^x + e^{-x})^2}$

**(D)** $\dfrac{4}{(e^x + e^{-x})^2}$ **(E)** $\dfrac{1}{e^{2x} + e^{-2x}}$

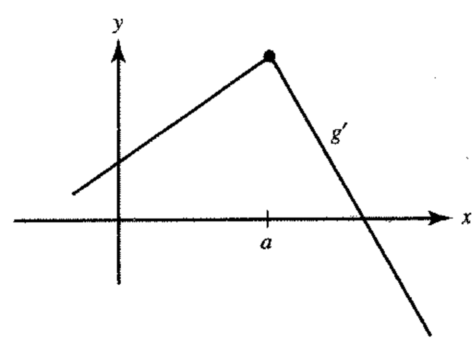
**21.** $y = \ln(x\sqrt{x^2+1})$

**(A)** $1 + \dfrac{x}{x^2+1}$ **(B)** $\dfrac{1}{x\sqrt{x^2+1}}$ **(C)** $\dfrac{2x^2+1}{x\sqrt{x^2+1}}$

**(D)** $\dfrac{2x^2+1}{x(x^2+1)}$ **(E)** none of these

**22.** The graph of $g'$ is shown here. Which of the following statements is (are) true of $g$ at $x = a$?

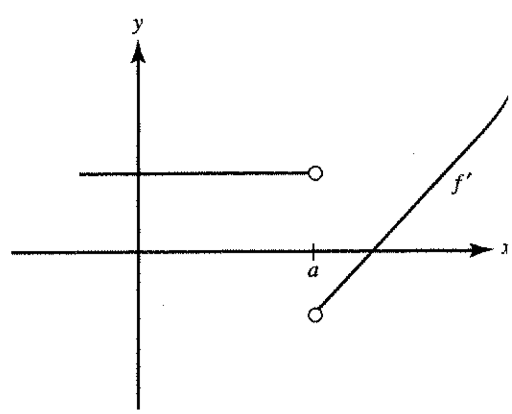
I. $g$ is continuous.
II. $g$ is differentiable.
III. $g$ is increasing.



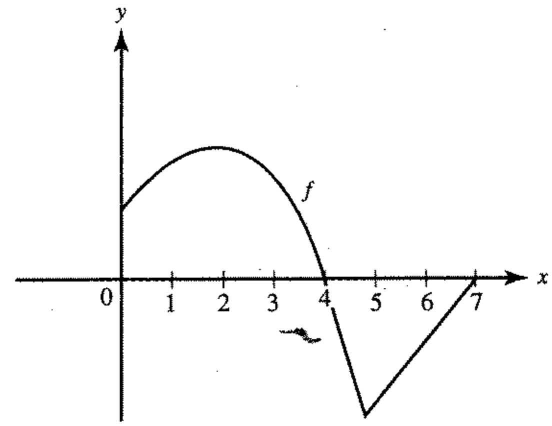

**(A)** I only **(B)** III only **(C)** I and III only
**(D)** II and III only **(E)** I, II, and III

**23.** A function $f$ has the derivative shown. Which of the following statements must be false?

**(A)** $f$ is continuous at $x = a$.
**(B)** $f(a) = 0$.
**(C)** $f$ has a vertical asymptote at $x = a$.
**(D)** $f$ has a jump discontinuity at $x = a$.
**(E)** $f$ has a removable discontinuity at $x = a$.



**24.** The function $f$ whose graph is shown has $f' = 0$ at $x =$



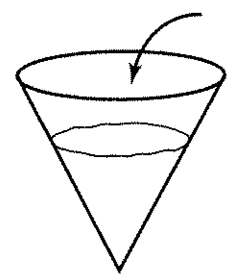

**(A)** 2 only
**(B)** 2 and 5
**(C)** 4 and 7
**(D)** 2, 4, and 7
**(E)** 2, 4, 5, and 7

**25.** A differentiable function $f$ has the values shown. Estimate $f'(1.5)$.

| $x$ | 1.0 | 1.2 | 1.4 | 1.6 |
|---|---|---|---|---|
| $f(x)$ | 8 | 10 | 14 | 22 |

**(A)** 8 **(B)** 12 **(C)** 18 **(D)** 40 **(E)** 80

**26.** Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then $h$ is



**(A)** constant
**(B)** linear and increasing
**(C)** linear and decreasing
**(D)** nonlinear and increasing
**(E)** nonlinear and decreasing

In Questions 27–33, find $\frac{dy}{dx}$.

**27.** $y = x^2 \sin \frac{1}{x}$ $(x \neq 0)$

**(A)** $2x \sin \frac{1}{x} - x^2 \cos \frac{1}{x}$ **(B)** $-\frac{2}{x} \cos \frac{1}{x}$ **(C)** $2x \cos \frac{1}{x}$

**(D)** $2x \sin \frac{1}{x} - \cos \frac{1}{x}$ **(E)** $-\cos \frac{1}{x}$

**28.** $y = \frac{1}{2 \sin 2x}$

**(A)** $-\csc 2x \cot 2x$ **(B)** $\frac{1}{4 \cos 2x}$ **(C)** $-4 \csc 2x \cot 2x$

**(D)** $\frac{\cos 2x}{2\sqrt{\sin 2x}}$ **(E)** $-\csc^2 2x$

**29.** $y = e^{-x} \cos 2x$

**(A)** $-e^{-x}(\cos 2x + 2 \sin 2x)$
**(B)** $e^{-x}(\sin 2x - \cos 2x)$
**(C)** $2e^{-x} \sin 2x$
**(D)** $-e^{-x}(\cos 2x + \sin 2x)$
**(E)** $-e^{-x} \sin 2x$

**30.** $y = \sec^2 \sqrt{x}$

**(A)** $\frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}}$ **(B)** $\frac{\tan \sqrt{x}}{\sqrt{x}}$ **(C)** $2 \sec \sqrt{x} \tan^2 \sqrt{x}$

**(D)** $\frac{\sec^2 \sqrt{x} \tan \sqrt{x}}{\sqrt{x}}$ **(E)** $2 \sec^2 \sqrt{x} \tan \sqrt{x}$

**31.** $y = x \ln^3 x$

**(A)** $\frac{3 \ln^2 x}{x}$ **(B)** $3 \ln^2 x$ **(C)** $3x \ln^2 x + \ln^3 x$

**(D)** $3(\ln x + 1)$ **(E)** none of these

**32.** $y = \frac{1+x^2}{1-x^2}$

**(A)** $-\frac{4x}{(1-x^2)^2}$ **(B)** $\frac{4x}{(1-x^2)^2}$ **(C)** $\frac{-4x^3}{(1-x^2)^2}$

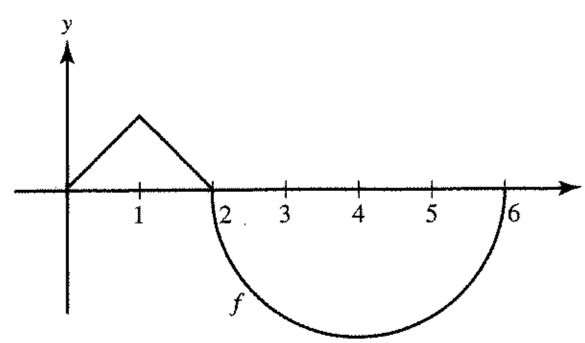
**(D)** $\frac{2x}{1-x^2}$ **(E)** $\frac{4}{1-x^2}$

**33.** $y = \sin^{-1} x - \sqrt{1-x^2}$

**(A)** $\frac{1}{2\sqrt{1-x^2}}$ **(B)** $\frac{2}{\sqrt{1-x^2}}$ **(C)** $\frac{1+x}{\sqrt{1-x^2}}$

**(D)** $\frac{x^2}{\sqrt{1-x^2}}$ **(E)** $\frac{1}{\sqrt{1+x}}$

Use the graph to answer Questions 34–36. It consists of two line segments and a semicircle.




**34.** $f'(x) = 0$ for $x =$

**(A)** 1 only
**(B)** 2 only
**(C)** 4 only
**(D)** 1 and 4
**(E)** 2 and 6

**35.** $f'(x)$ does not exist for $x =$

**(A)** 1 only **(B)** 2 only **(C)** 1 and 2
**(D)** 2 and 6 **(E)** 1, 2, and 6

**36.** $f'(5) =$

**(A)** $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 1 **(D)** 2 **(E)** $\sqrt{3}$

**37.** At how many points on the interval $[-5,5]$ is a tangent to $y = x + \cos x$ parallel to the secant line?

**(A)** none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

**38.** From the values of $f$ shown, estimate $f'(2)$.

| $x$ | 1.92 | 1.94 | 1.96 | 1.98 | 2.00 |
|---|---|---|---|---|---|
| $f(x)$ | 6.00 | 5.00 | 4.40 | 4.10 | 4.00 |

**(A)** $-0.10$ **(B)** $-0.20$ **(C)** $-5$ **(D)** $-10$ **(E)** $-25$

**39.** Using the values shown in the table for Question 38, estimate $(f^{-1})'(4)$.

**(A)** $-0.2$ **(B)** $-0.1$ **(C)** $-5$ **(D)** $-10$ **(E)** $-25$

**40.** The "left half" of the parabola defined by $y = x^2 - 8x + 10$ for $x \le 4$ is a one-to-one function; therefore its inverse is also a function. Call that inverse $g$. Find $g'(3)$.

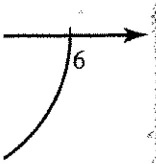
**(A)** $-\frac{1}{2}$ **(B)** $-\frac{1}{6}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{2}$

**41.** For $f(x) = 5^x$, what is the estimate of $f'(2)$ obtained by using the symmetric difference quotient with $h = 0.03$?

**(A)** 25.029 **(B)** 40.236 **(C)** 40.252 **(D)** 41.223 **(E)** 80.503

**42.** If $f$ is differentiable and difference quotients overestimate the slope of $f$ at $x = a$ for all $h > 0$, which must be true?

**(A)** $f'(a) > 0$ **(B)** $f'(a) < 0$ **(C)** $f''(a) > 0$
**(D)** $f''(a) < 0$ **(E)** none of these



In each of Questions 43–46, $y$ is a differentiable function of $x$. Choose the alternati that is the derivative $\frac{dy}{dx}$.

**43.** $x^3 - xy + y^3 = 1$

**(A)** $\dfrac{3x^2}{x - 3y^2}$ **(B)** $\dfrac{3x^2 - 1}{1 - 3y^2}$ **(C)** $\dfrac{y - 3x^2}{3y^2 - x}$

**(D)** $\dfrac{3x^2 + 3y^2 - y}{x}$ **(E)** $\dfrac{3x^2 + 3y^2}{x}$

**44.** $x + \cos(x + y) = 0$

**(A)** $\csc(x + y) - 1$ **(B)** $\csc(x + y)$ **(C)** $\dfrac{x}{\sin(x + y)}$

**(D)** $\dfrac{1}{\sqrt{1 - x^2}}$ **(E)** $\dfrac{1 - \sin x}{\sin y}$

**45.** $\sin x - \cos y - 2 = 0$

**(A)** $-\cot x$ **(B)** $-\cot y$ **(C)** $\dfrac{\cos x}{\sin y}$

**(D)** $-\csc y \cos x$ **(E)** $\dfrac{2 - \cos x}{\sin y}$

**46.** $3x^2 - 2xy + 5y^2 = 1$

**(A)** $\dfrac{3x + y}{x - 5y}$ **(B)** $\dfrac{y - 3x}{5y - x}$ **(C)** $3x + 5y$

**(D)** $\dfrac{3x + 4y}{x}$ **(E)** none of these

Individual instructions are given in full for each of the remaining questions of this s

***47.** If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then, when $t = 1$, $\frac{d^2y}{dx^2}$ is

**(A)** 1 **(B)** $-1$ **(C)** 0 **(D)** 3 **(E)** $\frac{1}{2}$

**48.** If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of $x$ for which the derivative equ zero is

**(A)** $\{1, 2\}$ **(B)** $\{0, -1, -2\}$ **(C)** $\{-1, +2\}$
**(D)** $\{0\}$ **(E)** $\{0, 1, 2\}$

**49.** If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

**(A)** $-32$ **(B)** $-16$ **(C)** $-4$ **(D)** $-2$ **(E)** $-\frac{1}{2}$

*An asterisk denotes a topic covered only in Calculus BC.

**50.** If $f(x) = \ln x^3$, then $f''(3)$ is

**(A)** $-\frac{1}{3}$ **(B)** $-1$ **(C)** $-3$ **(D)** $1$ **(E)** none of these

**51.** If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

**(A)** $0$ **(B)** $\frac{1}{5}$ **(C)** $-5$ **(D)** $-\frac{1}{5}$ **(E)** nonexistent

**52.** If $y = a \sin ct + b \cos ct$, where $a$, $b$, and $c$ are constants, then $\frac{d^2y}{dt^2}$ is

**(A)** $ac^2(\sin t + \cos t)$ **(B)** $-c^2y$ **(C)** $-ay$
**(D)** $-y$ **(E)** $a^2c^2 \sin ct - b^2c^2 \cos ct$

**53.** If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, then $(f \circ g)'(3)$ equals

**(A)** $0$ **(B)** $1$ **(C)** $6$ **(D)** $9$ **(E)** none of these

**54.** If $f(x) = 5^x$ and $5^{1.002} \approx 5.016$, which is closest to $f'(1)$?

**(A)** 0.016 **(B)** 1.0 **(C)** 5.0 **(D)** 8.0 **(E)** 32.0

**55.** If $f(x) = \frac{x}{(x-1)^2}$, then the set of $x$'s for which $f'(x)$ exists is

**(A)** all reals
**(B)** all reals except $x = 1$ and $x = -1$
**(C)** all reals except $x = -1$
**(D)** all reals except $x = \frac{1}{3}$ and $x = -1$
**(E)** all reals except $x = 1$

**56.** If $y = e^x(x - 1)$, then $y''(0)$ equals

**(A)** $-2$ **(B)** $-1$ **(C)** $0$ **(D)** $1$ **(E)** none of these

**57.** If $y = \sqrt{x^2 + 1}$, then the derivative of $y^2$ with respect to $x^2$ is

**(A)** $1$ **(B)** $\frac{x^2+1}{2x}$ **(C)** $\frac{x}{2(x^2+1)}$ **(D)** $\frac{2}{x}$ **(E)** $\frac{x^2}{x^2+1}$

**58.** If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

**(A)** $\frac{-\sqrt{x}}{(x^2+1)^2}$ **(B)** $-(x+1)^{-2}$ **(C)** $\frac{-2x}{(x^2+1)^2}$

**(D)** $\frac{1}{(x+1)^2}$ **(E)** $\frac{1}{2\sqrt{x}(x+1)}$

***59.** If $x = e^{\theta}\cos\theta$ and $y = e^{\theta}\sin\theta$, then, when $\theta = \frac{\pi}{2}$, $\frac{dy}{dx}$ is

**(A)** 1 **(B)** 0 **(C)** $e^{\pi/2}$ **(D)** nonexistent **(E)** $-1$

***60.** If $x = \cos t$ and $y = \cos 2t$, then $\frac{d^2y}{dx^2}$ $(\sin t \neq 0)$ is

**(A)** $4\cos t$ **(B)** 4 **(C)** $\frac{4y}{x}$ **(D)** $-4$ **(E)** $-4\cot t$

**61.** If $y = x^2 + x$, then the derivative of $y$ with respect to $\frac{1}{1-x}$ is

**(A)** $(2x+1)(x-1)^2$ **(B)** $\frac{2x+1}{(1-x)^2}$ **(C)** $2x+1$

**(D)** $\frac{3-x}{(1-x)^3}$ **(E)** none of these

**62.** $\lim_{h\to 0} \frac{(1+h)^6-1}{h}$ is

**(A)** 0 **(B)** 1 **(C)** 6 **(D)** $\infty$ **(E)** nonexistent

**63.** $\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h}$ is

**(A)** 0 **(B)** $\frac{1}{12}$ **(C)** 1 **(D)** 192 **(E)** $\infty$

**64.** $\lim_{h\to 0} \frac{\ln(e+h)-1}{h}$ is

**(A)** 0 **(B)** $\frac{1}{e}$ **(C)** 1 **(D)** $e$ **(E)** nonexistent

**65.** $\lim_{x\to 0} \frac{\cos x-1}{x}$ is

**(A)** $-1$ **(B)** 0 **(C)** 1 **(D)** $\infty$ **(E)** none of these

**66.** The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

**(A)** $f(0)$ is not defined **(B)** $f(x)$ is not continuous on $[-8, 8]$
**(C)** $f'(-1)$ does not exist **(D)** $f(x)$ is not defined for $x < 0$
**(E)** $f'(0)$ does not exist

**67.** If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

**(A)** $f(x)$ must be identically zero
**(B)** $f'(x)$ may be different from zero for all $x$ on $[a, b]$
**(C)** there exists at least one number $c$, $a < c < b$, such that $f'(c) = 0$
**(D)** $f'(x)$ must exist for every $x$ on $(a, b)$
**(E)** none of the preceding is true

*An asterisk denotes a topic covered only in Calculus BC.

**68.** If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \leqslant x \leqslant \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line?

**(A)** 1 **(B)** $-1$ **(C)** $\sqrt{2}$ **(D)** 0 **(E)** nowhere

**69.** If $h$ is the inverse function of $f$ and if $f(x) = \frac{1}{x}$, then $h'(3) =$

**(A)** $-9$ **(B)** $-\frac{1}{9}$ **(C)** $\frac{1}{9}$ **(D)** 3 **(E)** 9

**70.** Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of $f$, then $h'(-1) =$

**(A)** $-1$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{3}$ **(D)** 1 **(E)** 3

**71.** Suppose $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

**(A)** equals $-\frac{1}{3}$ **(B)** equals $-\frac{1}{4}$ **(C)** equals $\frac{1}{4}$

**(D)** equals $\frac{1}{3}$ **(E)** cannot be determined

**72.** If $f(x) = x^3 - 3x^2 + 8x + 5$ and $g(x) = f^{-1}(x)$, then $g'(5) =$

**(A)** 8 **(B)** $\frac{1}{8}$ **(C)** 1 **(D)** $\frac{1}{53}$ **(E)** 53

***73.** $\lim\limits_{x\to\infty} \frac{e^x}{x^{50}}$ equals

**(A)** 0 **(B)** 1 **(C)** $\frac{1}{50!}$ **(D)** $\infty$ **(E)** none of these

**74.** Suppose $\lim\limits_{x\to 0} \frac{g(x)-g(0)}{x} = 1$. It follows necessarily that

**(A)** $g$ is not defined at $x = 0$
**(B)** $g$ is not continuous at $x = 0$
**(C)** the limit of $g(x)$ as $x$ approaches 0 equals 1
**(D)** $g'(0) = 1$
**(E)** $g'(1) = 0$

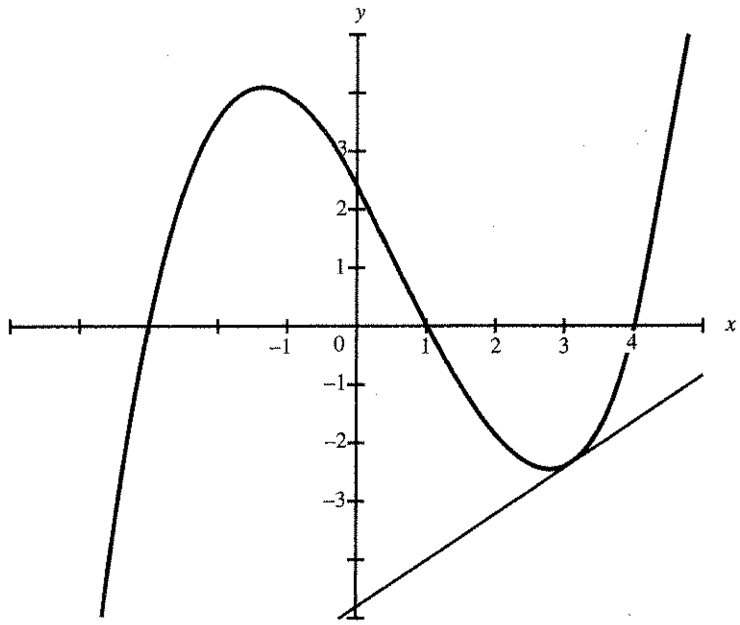
**75.** If $\sin(xy) = x$, then $\frac{dy}{dx} =$

**(A)** $\sec(xy)$ **(B)** $\frac{\sec(xy)}{x}$ **(C)** $\frac{\sec(xy) - y}{x}$

**(D)** $-\frac{1+\sec(xy)}{x}$ **(E)** $\sec(xy) - 1$

*An asterisk denotes a topic covered only in Calculus BC.

Use this graph of $y = f(x)$ for Questions 76 and 77.




**76.** $f'(3)$ is most closely approximated by

**(A)** 0.3 **(B)** 0.8 **(C)** 1.5 **(D)** 1.8 **(E)** 2

**77.** The rate of change of $f(x)$ is least at $x \simeq$

**(A)** –3 **(B)** –1.3 **(C)** 0 **(D)** 0.7 **(E)** 2.7

Use the following definition of the *symmetric difference quotient for* $f'(x_0)$ fo Questions 78–81: for small values of $h$,

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}.$$

**78.** $f'(x_0)$ equals the exact value of the derivative at $x = x_0$

**(A)** only when $f$ is linear
**(B)** whenever $f$ is quadratic
**(C)** if and only if $f'(x_0)$ exists
**(D)** whenever $|h| < 0.001$.
**(E)** none of these

**79.** To how many places is the symmetric difference accurate when it is used to approxima $f'(0)$ for $f(x) = 4^x$ and $h = 0.08$?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

**80.** To how many places is $f'(x_0)$ accurate when it is used to approximate $f'(0)$ fo $f(x) = 4^x$ and $h = 0.001$?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

**81.** The value of $f'(0)$ obtained using the symmetric difference quotient with $f(x) = |x|$ and $h = 0.001$ is

**(A)** $-1$ **(B)** 0 **(C)** $\pm 1$ **(D)** 1 **(E)** indeterminate

**82.** If $\frac{d}{dx} f(x) = g(x)$ and $h(x) = \sin x$, then $\frac{d}{dx} f(h(x))$ equals

**(A)** $g(\sin x)$ **(B)** $\cos x \cdot g(x)$ **(C)** $g'(x)$
**(D)** $\cos x \cdot g(\sin x)$ **(E)** $\sin x \cdot g(\sin x)$

**83.** $\lim\limits_{x \to 0} \frac{\sin 2x}{x}$ is

**(A)** 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** 0 **(E)** $\infty$

**84.** $\lim\limits_{x \to 0} \frac{\sin 3x}{\sin 4x}$ is

**(A)** 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{4}$ **(D)** 0 **(E)** nonexistent

**85.** $\lim\limits_{x \to 0} \frac{1 - \cos x}{x}$ is

**(A)** nonexistent **(B)** 1 **(C)** 2 **(D)** $\infty$ **(E)** none of these

**86.** $\lim\limits_{x \to 0} \frac{\tan \pi x}{x}$ is

**(A)** $\frac{1}{\pi}$ **(B)** 0 **(C)** 1 **(D)** $\pi$ **(E)** $\infty$

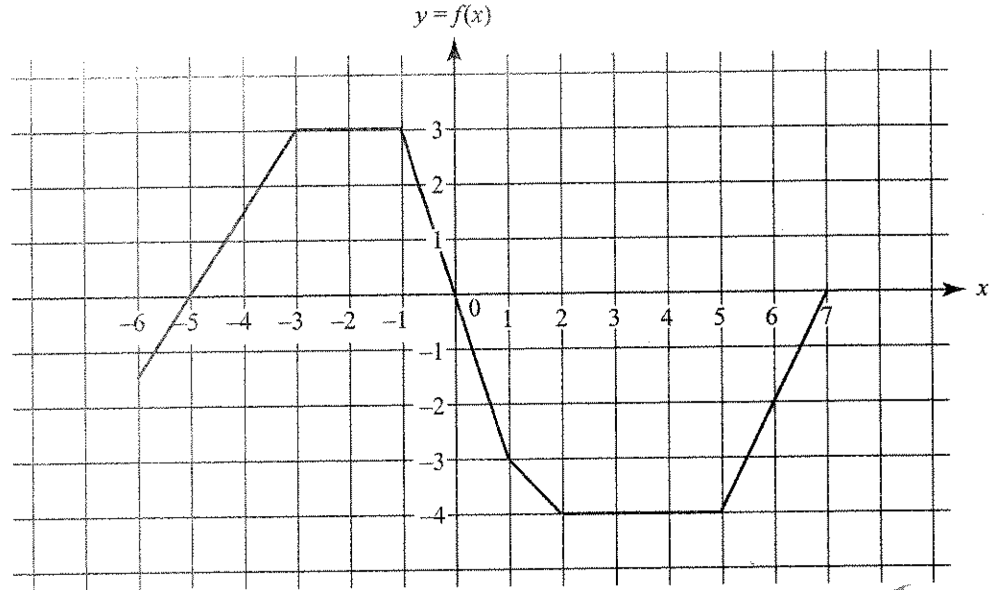
**87.** $\lim\limits_{x \to \infty} x^2 \sin \frac{1}{x}$

**(A)** is 1 **(B)** is 0 **(C)** is $\infty$
**(D)** oscillates between $-1$ and 1 **(E)** is none of these

**88.** Let $f(x) = 3^x - x^3$. The tangent to the curve is parallel to the secant through (0,1) and (3,0) for $x$ equal

**(A)** only to 0.984 **(B)** only to 1.244 **(C)** only to 2.727
**(D)** to 0.984 and 2.804 **(E)** to 1.244 and 2.727

Questions 89–93 are based on the following graph of $f(x)$, sketched on $-6 \leqslant x \leqslant$ Assume the horizontal and vertical grid lines are equally spaced at unit intervals.




**89.** On the interval $1 < x < 2, f(x)$ equals

**(A)** $-x-2$ **(B)** $-x-3$ **(C)** $-x-4$ **(D)** $-x+2$ **(E)** $x-$

**90.** Over which of the following intervals does $f'(x)$ equal zero?

I. $(-6,-3)$ II. $(-3,-1)$ III. $(2,5)$

**(A)** I only **(B)** II only **(C)** I and II only
**(D)** I and III only **(E)** II and III only

**91.** How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

**(A)** none **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

**92.** For $-6 < x -3, f'(x)$ equals

**(A)** $-\frac{3}{2}$ **(B)** $-1$ **(C)** $1$ **(D)** $\frac{3}{2}$ **(E)** $2$

**93.** Which of the following statements about the graph of $f'(x)$ is false?

**(A)** It consists of six horizontal segments.
**(B)** It has four jump discontinuities.
**(C)** $f'(x)$ is discontinuous at each $x$ in the set $\{-3,-1,1,2,5\}$.
**(D)** $f'(x)$ ranges from $-3$ to $2$.
**(E)** On the interval $-1 < x < 1, f'(x) = -3$.

***94.** The graph in the $xy$-plane represented by $x = 3 + 2 \sin t$ and $y = 2 \cos t - 1$, for $-\pi \leq t \leq \pi$, is

**(A)** a semicircle **(B)** a circle **(C)** an ellipse
**(D)** half of an ellipse **(E)** a hyperbola

**95.** $\lim\limits_{x \to 0} \dfrac{\sec x - \cos x}{x^2}$ equals

**(A)** 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** none of these

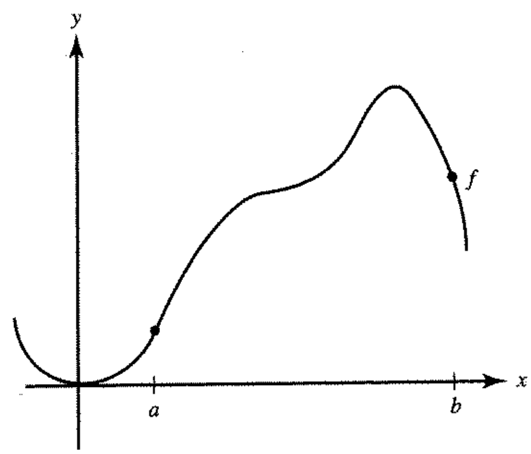
**96.** The table gives the values of a function $f$ that is differentiable on the interval $[0,1]$:

| $x$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 |
|---|---|---|---|---|---|---|
| $f(x)$ | 0.171 | 0.288 | 0.357 | 0.384 | 0.375 | 0.336 |

The best approximation of $f'(0.10)$ according to this table is

**(A)** 0.12 **(B)** 1.08 **(C)** 1.17 **(D)** 1.77 **(E)** ~~2.88~~

**97.** At how many points on the interval $[a,b]$ does the function graphed satisfy the Mean Value Theorem?




**(A)** none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

In each of Questions 98–101 a pair of equations that represents a curve parametrically is given. Choose the alternative that is the derivative $\dfrac{dy}{dx}$.

***98.** $x = t - \sin t$ and $y = 1 - \cos t$

**(A)** $\dfrac{\sin t}{1 - \cos t}$ **(B)** $\dfrac{1 - \cos t}{\sin t}$ **(C)** $\dfrac{\sin t}{\cos t - 1}$

**(D)** $\dfrac{1 - x}{y}$ **(E)** $\dfrac{1 - \cos t}{t - \sin t}$

*An asterisk denotes a topic covered only in Calculus BC.