remember: $y = f(x)$

1. $x^2 + y^2 = 36$

$$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}\left[36\right]$$

$$2x + 2yy' = 0$$

$$\frac{2yy'}{2y} = \frac{-2x}{2y}$$

$$y' = -\frac{x}{y}$$

3. $x^{1/2} + y^{1/2} = 9$

$$\frac{d}{dx}\left[x^{1/2}+y^{1/2}\right] = \frac{d}{dx}\left[9\right]$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$\frac{\cancel{\frac{1}{2}}\cancel{y^{-1/2}}y'}{\cancel{y^{-1/2}}} = \frac{-\cancel{\frac{1}{2}}x^{-1/2}}{y^{-1/2}}$$

$$y' = \frac{-x^{-1/2}}{y^{-1/2}} = \frac{-\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{y}}} = -\frac{\sqrt{y}}{\sqrt{x}} = -\sqrt{\frac{y}{x}}$$

5. $x^3 - xy + y^2 = 4$

$$\frac{d}{dx}\left(x^3-xy+y^2\right] = \frac{d}{dx}\left[4\right]$$

$$3x^2 - (xy'+y) + 2yy' = 0$$

$$3x^2 - xy' - y + 2yy' = 0$$

$$2yy' - xy' = y - 3x^2$$

$$\frac{y'(2y-x)}{2y-x} = \frac{y-3x^2}{2y-x}$$

$$\boxed{y' = \frac{y-3x^2}{2y-x}}$$

7. $x^3y^3 - y = x$

$$\frac{d}{dx}\left[x^3y^3 - y\right] = \frac{d}{dx}\left(x\right]$$

$$x^3\cdot 3y^2y' + y^3\cdot 3x^2 - y' = 1$$

$$3x^3y^2y' + 3x^2y^3 - y' = 1$$

$$3x^3y^2y' - y' = 1 - 3x^2y^3$$

$$\frac{y'(3x^3y^2-1)}{3x^3y^2-1} = \frac{1-3x^2y^3}{3x^3y^2-1}$$

$$\boxed{y' = \frac{1-3x^2y^3}{3x^3y^2-1}}$$

9. $x^3 - 3x^2y + 2xy^2 = 12$

$$\frac{d}{dx}\left[x^3-3x^2y+2xy^2\right] = \frac{d}{dx}\left(12\right]$$

$$3x^2 - (3x^2y' + y\cdot 6x) + 2x\cdot 2yy' + y^2\cdot 2 = 0$$

$$3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 = 0$$

$$4xyy' - 3x^2y' = 6xy - 3x^2 - 2y^2$$

$$\frac{y'(4xy-3x^2)}{4xy-3x^2} = \frac{6xy-3x^2-2y^2}{4xy-3x^2}$$

$$\boxed{y' = \frac{6xy-3x^2-2y^2}{4xy-3x^2}}$$

11. $\sin x + 2\cos 2y = 1$

$$\frac{d}{dx}\left(\sin x + 2\cos 2y\right] = \frac{d}{dx}\left[1\right]$$

$$\cos x + 4(-\sin 2y)\,y' = 0$$

$$\cos x - 4\sin 2y\cdot y' = 0$$

$$\cancel{+}4\sin 2y\cdot y' = \cancel{+}\cos x$$

$$\boxed{y' = \frac{\cos x}{4\sin 2y}}$$

13. $\sin x = x(1 + \tan y)$

$\frac{d}{dx}[\sin x] = \frac{d}{dx}[x(1+\tan y)]$

$\cos x = x(\sec^2 y \cdot y') + (1+\tan y)$

$\cos x = x\sec^2 y \cdot y' + 1 + \tan y$

$\cos x - 1 - \tan y = x\sec^2 y \cdot y'$

$$\boxed{y' = \frac{\cos x - 1 - \tan y}{x\sec^2 y}}$$

15. $y = \sin(xy)$

$\frac{d}{dx}[y] = \frac{d}{dx}[\sin(xy)]$

$y' = \cos(xy)(xy' + y)$

$y' = \cos(xy)\,xy' + \cos(xy)y$

$y' - xy'\cos(xy) = y\cos(xy)$

$y'(1 - x\cos(xy)) = y\cos(xy)$

$$\boxed{y' = \frac{y\cos(xy)}{1 - x\cos(xy)}}$$

21. $xy = 4$ $(-4, -1)$

$\frac{d}{dx}[xy] = \frac{d}{dx}[4]$

$xy' + y = 0$

$-4y' - 1 = 0$

$\frac{-4y'}{-4} = \frac{1}{-4}$

$$\boxed{y' = -\frac{1}{4}}$$

23. $y^2 = \frac{x^2 - 4}{x^2 + 4}$ $(2, 0)$

$\frac{d}{dx}[y^2] = \frac{d}{dx}\left[\frac{x^2-4}{x^2+4}\right]$

$2y \cdot y' = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2}$

$0 = \frac{32}{64} = \frac{1}{2}$

$0 \neq \frac{1}{2}$ no solutions

25. $x^{2/3} + y^{2/3} = 5$ $(8, 1)$

$\uparrow$ $\uparrow$
x y

$\frac{d}{dx}[x^{2/3} + y^{2/3}] = \frac{d}{dx}[5]$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$

$\frac{2}{3}\left(\frac{1}{2}\right) + \frac{2}{3}(1)y' = 0$

$\frac{1}{3} + \frac{2}{3}y' = 0$

$\frac{3}{2} \cdot \frac{2}{3}y' = -\frac{1}{3} \cdot \frac{3}{2}$

$$\boxed{y' = -1/2}$$

27. $\tan(x + y) = x$ $(0, 0)$

$\uparrow$ $\uparrow$
x y

$\frac{d}{dx}[\tan(x+y)] = \frac{d}{dx}[x]$

$\sec^2(x+y)(1+y') = 1$

$\sec^2(0)(1+y') = 1$

1

$\cancel{\left(\frac{1}{\cos 0}\right)^2}(1+y') = 1$

$1 + y' = 1$

$$\boxed{y' = 0}$$

35.

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d^2y}{dx^2}$$

$$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}[36]$$

$$2x+2yy'=0$$

$$2yy'=-2x$$

$$y' = \frac{-2x}{2y} = \frac{-x}{y}$$

$$\frac{d}{dx}\left[\frac{-x}{y}\right] = \frac{y(-1)-(-x)y'}{y^2}$$

$$= \frac{xy'-y}{y^2}$$

$$= \frac{x\left(-\frac{x}{y}\right)-y}{y^2}$$

$$= \left(\frac{-\frac{x^2}{y}-y}{y^2}\right)\frac{y}{y} = \frac{-x^2-y^2}{y^3}$$

$$= \frac{-1(x^2+y^2)}{y^3} = \boxed{\frac{-36}{y^3}}$$

37. $x^2 - y^2 = 16$

$$\frac{d}{dx}\left[x^2-y^2\right] = \frac{d}{dx}[16]$$

$$2x-2yy'=0$$

$$\frac{2x}{2y} = \frac{2yy'}{2y}$$

$$=$$

$$\frac{d}{dx}\left[\frac{x}{y}\right] = \frac{y-xy'}{y^2} \quad \frac{x}{y}$$

$$= \frac{y-\frac{x^2}{y}}{y^2}\,\frac{y}{y} = \frac{y^2-x^2}{y^3} = \frac{-1(x^2-y^2)}{y^3} = \boxed{\frac{-16}{y^3}}$$

39.

$$\frac{d}{dx}\left[y^2\right] = \frac{d}{dx}\left[x^3\right]$$

$$2yy' = 3x^2$$

$$=$$

$$\frac{d}{dx}\left[\frac{3x^2}{2y}\right] = \frac{2y\cdot 6x - 3x^2\cdot 2y'}{4y^2}$$

$$= \frac{12xy-6x^2y'}{4y^2} = \frac{12xy-6x^2\left(\frac{3x^2}{2y}\right)}{4y^2}$$

$$= \frac{12xy-\frac{18x^4}{2y}}{4y^2} = \frac{12xy-\frac{9x^4}{y}}{4y^2}\,\frac{y}{y}$$

$$= \frac{12xy^2-9x^4}{4y^3}$$

$$= \frac{12x\cdot x^3-9x^4}{4(x^3)y}$$

$$= \frac{12x^4-9x^4}{4x^3y} = \frac{3x^4}{4x^3y}$$

$$= \boxed{\frac{3x}{4y}}$$

41. $\sqrt{x} + \sqrt{y} = 4, \quad (9, 1)$

$\frac{d}{dx}\left[x^{1/2} + y^{1/2}\right] = \frac{d}{dx}[4]$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$

$\frac{1}{2}\frac{1}{\sqrt{x}} + \frac{1}{2}\frac{1}{\sqrt{y}}y' = 0$

$\frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\left(\frac{1}{1}\right)y' = 0$

$\frac{1}{6} + \frac{1}{2}y' = 0$

$2\cdot\frac{1}{2}y' = -\frac{1}{6}\cdot 2$

$y' = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = -\frac{1}{3}(x - 9)}$

$y = -\frac{1}{3}x + 3 + 1$

$\boxed{y = -\frac{1}{3}x + 4}$

43. $x^2 + y^2 = 25$
$(4, 3), (-3, 4)$

$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[25]$

$2x + 2yy' = 0$

$2yy' = -2x$

$y' = -\frac{2x}{2y} = -\frac{x}{y}$

use $(-3,4)$:

$m_{tan} = -\frac{x}{y} = \frac{3}{4}$

$m_{norm} = -\frac{4}{3}$

Use $(4,3)$:

$m_{tan} = -\frac{x}{y} = -\frac{4}{3}$

$m_{norm} = \frac{3}{4}$

$y - y_1 = m(x - x_1)$
$y - 3 = -\frac{4}{3}(x - 4)$
equation of tangent line

$y - y_1 = m(x - x_1)$
$y - 3 = \frac{3}{4}(x - 4)$
equation of normal line

$y - y_1 = m(x - x_1)$
$y - 4 = \frac{3}{4}(x + 3)$
equation of tangent line

$y - y_1 = m(x - x_1)$
$y - 4 = -\frac{4}{3}(x + 3)$
equation of normal line

Implicit plot: