Given the curve $x + xy + 2y^2 = 6$.

(a) Find an expression for the slope of the curve at any point (x, y) on the curve.

(b) Write an equation for the line tangent to the curve at the point $(2,1)$.

(c) Find the coordinates of all other points on this curve with slope equal to the slope at $(2,1)$.

$$\frac{d}{dx}\left[x + xy + 2y^2\right] = \frac{d}{dx}[6]$$

$$1 + xy' + y + 4yy' = 0$$

$$xy' + 4yy' = -1 - y$$

$$\frac{y'(x+4y)}{x+4y} = \frac{-1-y}{x+4y}$$

$$\boxed{y' = \frac{-1-y}{x+4y}}$$

$m = -\frac{1}{3}$ $x_1 = 2$ $y_1 = 1$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 1 = -\frac{1}{3}(x - 2)}$$

$$\frac{-1-y}{x+4y} = -\frac{1}{3}$$

$$\frac{-1-y}{x+4y} = -\frac{1}{3}$$

$$+\frac{(1+y)}{x+4y} = +\frac{1}{3}$$

$$3(1+y) = x + 4y$$

$$3 + 3y = x + 4y$$

$$y = 3 - x$$

$$x + xy + 2y^2 = 6$$

$$x + x(3-x) + 2(3-x)^2 = 6$$

$$x + 3x - x^2 + 2(9 - 6x + x^2) = 6$$

$$4x - x^2 + 18 - 12x + 2x^2 - 6 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$x = 2$ $x = 6$

$y = 3 - 2 = 1$ $y = 3 - 6 = -3$

$(2,1)$ $(6,-3)$

Given the curve $x^2 - xy + y^2 = 9$.

(a) Write a general expression for the slope of the curve.

(b) Find the coordinates of the points on the curve where the tangents are vertical.

$2y - x = 0$

$x = 2y$

(c) At the point $(0,3)$ find the rate of change in the slope of the curve with respect to x.

$$\frac{d}{dx}\left[x^2 - xy + y^2\right] = \frac{d}{dx}[9]$$

$$2x - (xy' + y) + 2yy' = 0$$

$$2x - xy' - y + 2yy' = 0$$

$$2yy' - xy' = y - 2x$$

$$y'(2y - x) = y - 2x$$

$$\boxed{y' = \frac{y-2x}{2y-x}}$$

$$(2y)^2 - (2y)y + y^2 = 9$$

$$4y^2 - 2y^2 + y^2 = 9$$

$$3y^2 = 9$$

$$y^2 = 3, \quad y = \pm\sqrt{3}$$

$(2\sqrt{3}, \sqrt{3})$ $\quad (-2\sqrt{3}, -\sqrt{3})$

$$\boxed{y' = \frac{y-2x}{2y-x}} \quad @ (0,3) \quad y' = \frac{3}{6} = \frac{1}{2}$$

$$\frac{d}{dx}\left[\frac{y-2x}{2y-x}\right] = \frac{(2y-x)(y'-2) - (y-2x)(2y'-1)}{(2y-x)^2}$$

$$= \frac{(2(3)-(0))(\frac{1}{2}-2) - ((3)-2(0))(2(\frac{1}{2})-1)}{(2(3)-(0))^2}$$

$$= \frac{6(-\frac{3}{2}) - \cancel{3(0)}}{36} = \frac{-9}{36} = \boxed{-\frac{1}{4}}$$

Consider the curve given by the equation $y^3 + 3x^2y + 13 = 0$.

(a) Find $\frac{dy}{dx}$.

$$m = \frac{-6(2)(-1)}{3(-1)^2 + 3(2)^2} = \frac{12}{3+12} = \frac{12}{15} = \frac{4}{5}$$

(b) Write an equation for the line tangent to the curve at the point $(2, -1)$.

$$\frac{d}{dx}\left[y^3 + 3x^2y + 13\right] = \frac{d}{dx}[0]$$

$$3y^2y' + 3x^2y' + 6xy = 0$$

$$y'(3y^2 + 3x^2) = -6xy$$

$$\boxed{y' = \frac{-6xy}{3y^2 + 3x^2}}$$

$m = \frac{4}{5} \quad x_1 = 2 \quad y_1 = -1$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y + 1 = \frac{4}{5}(x - 2)}$$