

# Integration by Substitution Part II

Note Title

12/7/2008

Evaluate  $\int_0^1 x(x^2+1)^3 dx$

$$\begin{aligned}u &= x^2 + 1 & u(1) &= (1)^2 + 1 = 2 \\ du &= 2x dx & u(0) &= (0)^2 + 1 = 1 \\ \frac{1}{2} du &= x dx\end{aligned}$$

$$\frac{1}{2} \int_1^2 u^3 du$$

$$\frac{1}{2} \cdot \frac{1}{4} [u^4]_1^2 = \frac{1}{8} [16 - 1] = \frac{15}{8}$$

Evaluate  $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$

$$\begin{aligned}u &= 2x - 1 & u(5) &= 9 \\ du &= 2 dx & u(1) &= 1 \\ \frac{1}{2} du &= dx\end{aligned}$$

$$\frac{1}{2} \int_1^9 x \cdot u^{-1/2} du$$

$$\begin{aligned}u &= 2x - 1 \\ u + 1 &= 2x \\ x &= \frac{u+1}{2} = \frac{1}{2}(u+1)\end{aligned}$$

$$\frac{1}{4} \int_1^9 (u+1) u^{-1/2} du$$

$$\frac{1}{4} \int_1^9 u^{1/2} + u^{-1/2} du = \frac{1}{4} \left[ \frac{2}{3} u^{3/2} + 2u^{1/2} \right]_1^9$$

$$= \frac{1}{4} \left[ (18 + 6) - \left( \frac{2}{3} + 2 \right) \right]$$

$$= \frac{1}{4} \left[ 24 - \frac{2}{3} - 2 \right]$$

$$= \frac{1}{4} \left[ 22 - \frac{2}{3} \right]$$

$$= \frac{1}{4} \left( \frac{64}{3} \right) = \frac{64}{12} = \frac{32}{6} = \frac{16}{3}$$

Evaluate:  $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$   $du = \sqrt{2x-1} = (2x-1)^{1/2}$   
 $du = \frac{1}{2}(2x-1)^{-1/2} \cdot 2 dx = \frac{dx}{\sqrt{2x-1}}$

$$\int_{u=1}^{u=3} x du$$

$$u(5) = 3 \quad u(1) = 1$$

$$\frac{1}{2} \int_1^3 u^2 + 1 du$$

$$u = \sqrt{2x-1}$$

$$u^2 = 2x-1$$

$$u^2 + 1 = 2x$$

$$x = \frac{1}{2}(u^2 + 1)$$

$$\frac{1}{2} \left[ \frac{u^3}{3} + u \right]_1^3 = \frac{1}{2} \left[ (9+3) - \left( \frac{1}{3} + 1 \right) \right]$$

$$= \frac{1}{2} \left[ 12 - \frac{1}{3} - 1 \right] = \frac{1}{2} \left[ 10 \frac{2}{3} \right] = \frac{1}{2} \left[ \frac{32}{3} \right] = \frac{16}{3}$$