Integration by Substitution   Part I

Find: $\int (x^2+1)^2 (2x)\,dx$ $u = x^2+1$
                                 $du = 2x\,dx$

$\int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(x^2+1)^3 + C$

Find: $\int 5\cos 5x\,dx$ $u = 5x$
                           $du = 5\,dx$

$\int \cos u\,du = \sin u + C = \sin 5x + C$

Find $\int x(x^2+1)^2\,dx$ $u = x^2+1$
                            $du = 2x\,dx$
                            $\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^2\,du = \frac{1}{2}\cdot\frac{1}{3}(x^2+1)^3 + C$

$= \frac{1}{6}(x^2+1)^3 + C$

Find: $\int \sqrt{2x-1}\,dx$ $u = 2x-1$
                              $du = 2\,dx$
                              $\frac{1}{2}du = dx$

$\frac{1}{2}\int u^{1/2}\,du$

$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right] + C = \frac{1}{3}(2x-1)^{3/2} + C$

Find: $\int x\sqrt{2x-1}\,dx$ $u = 2x-1 \leftarrow$ $u+1 = 2x$
                                $du = 2\,dx$ $x = \frac{u+1}{2}$
                                $\frac{1}{2}du = dx$

$= \frac{1}{2}\int x\, u^{1/2}\,du$

$= \frac{1}{2}\int \frac{u+1}{2}\,u^{1/2}\,du = \frac{1}{4}\int (u+1)u^{1/2}\,du = \frac{1}{4}\int u^{3/2} + u^{1/2}\,du$

$\frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] = \frac{1}{4}\left[\frac{2}{5}(2x-1)^{5/2} + \frac{2}{3}(2x-1)^{3/2}\right] + C$

Find: $\int \sin^2 3x \cos 3x \, dx$

$u = \sin 3x$
$du = 3\cos 3x \, dx$
$\frac{1}{3} du = \cos 3x \, dx$

$\frac{1}{3} \int u^2 \, du = \frac{1}{9} u^3 + C = \boxed{\frac{1}{9}(\sin^3 3x) + C}$

$\frac{d}{dx}\left[\frac{1}{9}\sin^3 3x + C\right] = \frac{1}{9} \cdot \cancel{3} \sin^2 3x \cos 3x \cdot \cancel{3} + 0$

$\qquad\qquad = \sin^2 3x \cos 3x$

a) $\int 3(3x-1)^4 \, dx$

$u = 3x - 1$
$du = 3 \, dx$

$\int u^4 \, du = \boxed{\frac{1}{5}(3x-1)^5 + C}$

b) $\int (2x+1)(x^2+x) \, dx$

$u = x^2 + x$
$du = 2x+1 \, dx$

$\int u \, du = \boxed{\frac{1}{2}(x^2+x)^2 + C}$

c. $\int 3x^2 \sqrt{x^3-2} \, dx$

$u = x^3 - 2$
$du = 3x^2 \, dx$

$\int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C = \boxed{\frac{2}{3}(x^3-2)^{3/2} + C}$

d. $\int \frac{4x}{(1-2x^2)^2} \, dx$

$u = 1 - 2x^2$
$du = -4x \, dx$

$\int \frac{du}{u^2} = \int u^{-2} \, du = -u^{-1} + C = \boxed{\frac{-1}{(1-2x^2)} + C}$

e. $\int \cos^2 x \sin x \, dx$

$u = \cos x$
$du = \sin x \, dx$

$\int u^2 \, du = \boxed{\frac{1}{3}(\cos^3 x) + C}$