In Exercises 3-8, find the indefinite integral.

3.
$$\int (2x^2 + x - 1) dx$$

4. $\int \frac{2}{\sqrt[3]{3x}} dx$

5.
$$\int \frac{x^3 + 1}{x^2} dx$$

6.
$$\int \frac{x^3 - 2x^2 + 1}{x^2} dx$$

7.
$$\int (4x - 3\sin x) dx$$

8.
$$\int (5\cos x - 2\sec^2 x) dx$$

- Find the particular solution of the differential equation f'(x) = −2x whose graph passes through the point (−1, 1).
- 10. Find the particular solution of the differential equation f''(x) = 6(x 1) whose graph passes through the point (2, 1) and is tangent to the line 3x y 5 = 0 at that point.
- 11. Velocity and Acceleration An airplane taking off from a runway travels 3600 feet before lifting off. If it starts from rest, moves with constant acceleration, and makes the run in 30 seconds, with what speed does it lift off?
- 12. Velocity and Acceleration The speed of a car traveling in a straight line is reduced from 45 to 30 miles per hour in a distance of 264 feet. Find the distance in which the car can be brought to rest from 30 miles per hour, assuming the same constant deceleration.
- Velocity and Acceleration A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second.
 - (a) How long will it take the ball to rise to its maximum height?
 - (b) What is the maximum height?
 - (c) When is the velocity of the ball one-half the initial velocity?
 - (d) What is the height of the ball when its velocity is one-half the initial velocity?
- Velocity and Acceleration Repeat Exercise 13 for an initial velocity of 40 meters per second.

In Exercises 27 and 28, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

27.
$$\int_0^5 (5 - |x - 5|) dx$$

28.
$$\int_{-4}^{4} \sqrt{16 - x^2} \, dx$$

In Exercises 29 and 30, use the given values to evaluate each definite integral.

29. If
$$\int_{2}^{6} f(x) dx = 10$$
 and $\int_{2}^{6} g(x) dx = 3$, find

(a)
$$\int_{2}^{\pi} [f(x) + g(x)] dx$$
.

(a)
$$\int_{2}^{6} [f(x) + g(x)] dx$$
. (b) $\int_{2}^{6} [f(x) - g(x)] dx$.

(c)
$$\int_{2}^{6} [2f(x) - 3g(x)] dx$$
. (d) $\int_{2}^{6} 5f(x) dx$.

(d)
$$\int_{2}^{6} 5f(x) dx$$

30. If
$$\int_0^3 f(x) dx = 4$$
 and $\int_3^6 f(x) dx = -1$, find

(a)
$$\int_0^6 f(x) \, dx.$$

(b)
$$\int_{a}^{3} f(x) dx.$$

(c)
$$\int_{4}^{4} f(x) dx.$$

(d)
$$\int_{0}^{6} -10 f(x) dx$$
.

In Exercises 33-40, use the Fundamental Theorem of Calculus to evaluate the definite integral.

33.
$$\int_0^4 (2+x) dx$$

34.
$$\int_{-1}^{1} (t^2 + 2) dt$$

35.
$$\int_{-1}^{1} (4t^3 - 2t) dt$$

36.
$$\int_{-2}^{2} (x^4 + 2x^2 - 5) dx$$

37.
$$\int_{4}^{9} x \sqrt{x} \, dx$$

38.
$$\int_{1}^{2} \left(\frac{1}{x^2} - \frac{1}{x^3} \right) dx$$

39.
$$\int_{0}^{3\pi/4} \sin\theta \, d\theta$$

40.
$$\int_{-t/4}^{\pi/4} \sec^2 t \, dt$$

In Exercises 41-46, sketch the graph of the region whose area is given by the integral, and find the area.

41.
$$\int_{1}^{3} (2x - 1) \, dx$$

42.
$$\int_{0}^{2} (x+4) dx$$

43.
$$\int_{3}^{4} (x^2 - 9) dx$$

44.
$$\int_{-1}^{2} (-x^2 + x + 2) dx$$

45.
$$\int_{0}^{1} (x - x^{3}) dx$$

46.
$$\int_0^1 \sqrt{x} (1-x) dx$$

In Exercises 47 and 48, sketch the region bounded by the graphs of the equations, and determine its area.

47.
$$y = \frac{4}{\sqrt{x}}$$
, $y = 0$, $x = 1$, $x = 9$

48.
$$y = \sec^2 x$$
, $y = 0$, $x = 0$, $x = \frac{\pi}{3}$

In Exercises 49 and 50, find the average value of the function over the interval. Find the values of x at which the function assumes its average value, and graph the function.

Function

Interval

49.
$$f(x) = \frac{1}{\sqrt{x}}$$

[4, 9]

50.
$$f(x) = x^3$$
 [0, 2]

In Exercises 51–54, use the Second Fundamental Theorem of Calculus to find F'(x).

51.
$$F(x) = \int_0^x t^2 \sqrt{1 + t^3} dt$$
 52. $F(x) = \int_1^x \frac{1}{t^2} dt$ **53.** $F(x) = \int_0^x (t^2 + 3t + 2) dt$ **54.** $F(x) = \int_0^x \csc^2 t dt$

4.5 In Exercises 55–68, find the indefinite integral.

65. $\int \tan^n x \sec^2 x \, dx, \qquad n \neq -1$

67. $\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x \, dx$

66. $\int \sec 2x \tan 2x \, dx$

68. $\int \cot^4 \alpha \csc^2 \alpha \, d\alpha$

55.
$$\int (x^2 + 1)^3 dx$$

56. $\int \left(x + \frac{1}{x}\right)^2 dx$

57. $\int \frac{x^2}{\sqrt{x^3 + 3}} dx$

58. $\int x^2 \sqrt{x^3 + 3} dx$

59. $\int x(1 - 3x^2)^4 dx$

60. $\int \frac{x + 3}{(x^2 + 6x - 5)^2} dx$

61. $\int \sin^3 x \cos x dx$

62. $\int x \sin 3x^2 dx$

63. $\int \frac{\sin \theta}{\sqrt{1 - \cos \theta}} d\theta$

64. $\int \frac{\cos x}{\sqrt{\sin x}} dx$

In Exercises 69-76, evaluate the definite integral. Use a graphing utility to verify your result.

69.
$$\int_{-1}^{2} x(x^2 - 4) dx$$

70.
$$\int_0^1 x^2 (x^3 + 1)^3 dx$$

71.
$$\int_0^3 \frac{1}{\sqrt{1+x}} \, dx$$

72.
$$\int_{3}^{6} \frac{x}{3\sqrt{x^2 - 8}} dx$$

73.
$$2\pi \int_{0}^{1} (y+1)\sqrt{1-y} \, dy$$

70.
$$\int_{0}^{1} x^{2}(x^{3} + 1)^{3} dx$$
72.
$$\int_{3}^{6} \frac{x}{3\sqrt{x^{2} - 8}} dx$$
74.
$$2\pi \int_{-1}^{0} x^{2} \sqrt{x + 1} dx$$

75.
$$\int_0^{\pi} \cos \frac{x}{2} dx$$

76.
$$\int_{-\pi/4}^{\pi/4} \sin 2x \, dx$$

4.6 In Exercises 81–84, use the Trapezoidal Rule and Simpson's Rule with n = 4, and use the integration capabilities of a graphing utility, to approximate the definite integral. Compare the results.

81.
$$\int_{1}^{2} \frac{1}{1+x^3} \, dx$$

82.
$$\int_0^1 \frac{x^{3/2}}{3-x^2} dx$$

83.
$$\int_{0}^{\pi/2} \sqrt{x} \cos x \, dx$$

84.
$$\int_{0}^{\pi} \sqrt{1 + \sin^2 x} \, dx$$