Ch.5 Sect 1

45. $g(x) = \ln x^2$

47. $y = (\ln x)^4$

49. $y = \ln\left(x\sqrt{x^2-1}\right)$

51. $f(x) = \ln\left(\dfrac{x}{x^2+1}\right)$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$

$\ln x^a = a\ln x$

$\rightarrow \ln(a\cdot b) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

45. $\frac{d}{dx}[\ln x^2] = \frac{d}{dx}[2\ln x] = \boxed{\frac{2}{x}}$

$\frac{d}{dx}[\ln x^2] = \frac{1}{x^2}\cdot 2x = \frac{2x}{x^2} = \frac{2}{x}$

47. $\frac{d}{dx}[(\ln x)^4] = \frac{d}{dx}[\ln^4 x] = 4\ln^3 x\cdot\frac{1}{x}$

$= \boxed{\frac{4\ln^3 x}{x}}$

49. $\frac{d}{dx}\left[\ln\left(x\sqrt{x^2-1}\right)\right]$

$= \frac{d}{dx}[\ln x] + \frac{d}{dx}\left[\ln(x^2-1)^{1/2}\right] = \frac{d}{dx}[\ln x] + \frac{d}{dx}\left[\frac{1}{2}\ln(x^2-1)\right]$

$= \frac{1}{x} + \frac{1}{\cancel{2}}\cdot\frac{1}{x^2-1}\cdot\cancel{2}x = \frac{1}{x} + \frac{x}{x^2-1} = \frac{x^2-1}{x(x^2-1)} + \frac{x^2}{x(x^2-1)} = \boxed{\frac{2x^2-1}{x(x^2-1)}}$

51. $\frac{d}{dx}\left[\ln\left(\frac{x}{x^2+1}\right)\right] = \frac{d}{dx}[\ln x - \ln(x^2+1)] = \frac{1}{x} - \frac{2x}{x^2+1}$

$= \frac{x^2+1}{x(x^2+1)} - \frac{2x^2}{x(x^2+1)} = \boxed{\frac{1-x^2}{x(x^2+1)}}$

53. $g(t) = \dfrac{\ln t}{t^2}$

55. $y = \ln(\ln x^2)$

57. $y = \ln\sqrt{\dfrac{x+1}{x-1}}$

53. $\frac{d}{dt}\left[\frac{\ln t}{t^2}\right] = \frac{t^2\cdot\frac{1}{t} - (\ln t)2t}{t^4} = \frac{t - 2t\ln t}{t^4}$

$= \frac{\cancel{t}(1-2\ln t)}{t^{\cancel{4}3}}$

$= \boxed{\frac{1-2\ln t}{t^3}}$

55. $\frac{d}{dx}[\ln(2\ln x)] =$

$\frac{1}{\cancel{2}\ln x}\cdot\frac{\cancel{2}}{x} = \boxed{\frac{1}{x\ln x}}$

57. $\frac{d}{dx}\left[\ln\sqrt{\frac{x+1}{x-1}}\right] = \frac{d}{dx}\left[\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)\right] = \frac{1}{2}\frac{d}{dx}[\ln(x+1) - \ln(x-1)]$

$= \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right]$

$= \frac{1}{2}\left[\frac{x-1}{(x+1)(x-1)} - \frac{x+1}{(x+1)(x-1)}\right]$

$= \frac{1}{\cancel{2}}\left[\frac{-\cancel{2}}{(x+1)(x-1)}\right] = \boxed{\frac{-1}{(x+1)(x-1)}}$

59. $f(x) = \ln\left(\dfrac{\sqrt{4+x^2}}{x}\right) = \ln\sqrt{4+x^2} - \ln x = \frac{1}{2}\ln(4+x^2) - \ln x$

$\frac{d}{dx}\left[\frac{1}{2}\ln(4+x^2) - \ln x\right] = \frac{1}{\cancel{2}}\,\frac{1}{4+x^2}\cdot\frac{\cancel{2}x}{1} - \frac{1}{x}$

$= \frac{x}{4+x^2} - \frac{1}{x} = \frac{x^2}{x(4+x^2)} - \frac{4+x^2}{x(4+x^2)}$

$= \boxed{\frac{-4}{x(4+x^2)}}$

39. $f(x) = e^{2x}$

41. $y = e^{-2x+x^2}$

43. $y = e^{\sqrt{x}}$

45. $g(t) = (e^{-t} + e^t)^3$

47. $y = \ln(e^{x^2})$

$\ln e = 1$

$\ln 1 = 0$

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}$

$x^a \cdot x^b = x^{a+b}$

39. $\frac{d}{dx}[e^{2x}] = e^{2x} \cdot 2 = \boxed{2e^{2x}}$

41. $\frac{d}{dx}\left[e^{-2x+x^2}\right] = e^{-2x+x^2}(-2+2x)$

$= \boxed{(2x-2)e^{-2x+x^2}}$

43. $\frac{d}{dx}\left[e^{\sqrt{x}}\right] = e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) = \boxed{\frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

45. $\frac{d}{dt}\left[(e^{-t}+e^t)^3\right] = 3(e^{-t}+e^t)^2\left[e^{-t}(-1) + e^t\right]$

$= \boxed{3(e^{-t}+e^t)^2(e^t - e^{-t})}$

47. $\frac{d}{dx}\left[\ln e^{x^2}\right] = \frac{d}{dx}\left[x^2 \cancel{\ln e}^{\,1}\right] = \boxed{2x}$

49. $y = \ln(1 + e^{2x})$

49) $\frac{d}{dx}\left[\ln(1+e^{2x})\right] = \frac{1}{1+e^{2x}}(e^{2x} \cdot 2) = \boxed{\frac{2e^{2x}}{1+e^{2x}}}$

51. $y = \dfrac{2}{e^x + e^{-x}}$

53. $y = x^2e^x - 2xe^x + 2e^x$

55. $f(x) = e^{-x}\ln x$

57. $y = e^x(\sin x + \cos x)$

51) $\frac{d}{dx}\left[\frac{2}{e^x+e^{-x}}\right] = \frac{-2\frac{d}{dx}[e^x+e^{-x}]}{(e^x+e^{-x})^2}$

$= \boxed{\frac{-2(e^x-e^{-x})}{(e^x+e^{-x})^2}}$

53)

$\frac{d}{dx}\left[x^2e^x - 2xe^x + 2e^x\right]$

$= x^2e^x + \cancel{2x}e^x - (\cancel{2x}e^x + \cancel{2e^x}) + \cancel{2e^x}$

$= \boxed{x^2e^x}$

55) $\frac{d}{dx}\left[e^{-x}\ln x\right] = e^{-x}\left(\frac{1}{x}\right) + \ln x(-e^{-x}) = \frac{e^{-x}}{x} - e^{-x}\ln x$

$= e^{-x}\left(\frac{1}{x} - \ln x\right)$

$= \boxed{e^{-x}\left(\frac{1-x\ln x}{x}\right)}$

57) $\frac{d}{dx}\left[e^x(\sin x + \cos x)\right] =$

$e^x(\cos x - \sin x) + e^x(\sin x + \cos x) = e^x(2\cos x) = \boxed{2e^x\cos x}$

41. $f(x) = 4^x$

43. $y = 5^{x-2}$

45. $g(t) = t^2 2^t$

Ch. 5 Sect 5

$\frac{d}{dx}[a^x] = a^x \ln a$

$\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$

$\frac{d}{dx}[\log_a x] = \frac{1}{x\ln a}$

$\frac{d}{dx}[\log_a u] = \frac{1}{u\ln a} \cdot \frac{du}{dx}$

41) $\frac{d}{dx}[4^x] = \boxed{4^x \ln 4}$

43) $\frac{d}{dx}[5^{x-2}] = 5^{x-2}(\ln 5) = \boxed{5^{x-2}\ln 5}$

45) $\frac{d}{dt}[t^2 2^t] = t^2 \cdot 2^t \ln 2 + 2t \cdot 2^t$

$= 2^t(t^2\ln 2 + 2t)$

$= \boxed{2^t t(t\ln 2 + 2)}$

47. $h(\theta) = 2^{-\theta}\cos \pi\theta$

47) $\frac{d}{d\theta}\left[2^{-\theta}\cos \pi\theta\right] = 2^{-\theta}(-\sin \pi\theta)(\pi)$

$+ \cos \pi\theta \cdot 2^{-\theta}\ln 2\,(-1)$

$= -2^{-\theta}\pi\sin \pi\theta - 2^{-\theta}\ln 2\cos \pi\theta$

$= \boxed{-2^{-\theta}(\pi\sin \pi\theta + \ln 2\cos \pi\theta)}$

49. $y = \log_3 x$

51. $f(x) = \log_2 \dfrac{x^2}{x-1}$

53. $y = \log_5 \sqrt{x^2 - 1}$

55. $g(t) = \dfrac{10 \log_4 t}{t}$

49) $\dfrac{d}{dx}[\log_3 x] = \boxed{\dfrac{1}{x \ln 3}}$

51) $\dfrac{d}{dx}\left[\log_2\left(\dfrac{x^2}{x-1}\right)\right] = \dfrac{d}{dx}\left[\log_2 x^2 - \log_2(x-1)\right]$

$= \dfrac{d}{dx}\left[2\log_2 x - \log_2(x-1)\right]$

$= \dfrac{2}{x \ln 2} - \dfrac{1}{(x-1)\ln 2}$

$= \dfrac{2(x-1)}{x(x-1)\ln 2} - \dfrac{x}{x(x-1)\ln 2}$

$= \boxed{\dfrac{x-2}{x(x-1)\ln 2}}$

$\dfrac{d}{dx}[\log_a x] = \dfrac{1}{x \ln a}$

$\dfrac{d}{dx}[\log_a u] = \dfrac{1}{u \ln a} \cdot \dfrac{du}{dx}$

53) $\dfrac{d}{dx}\left[\log_5 (x^2-1)^{1/2}\right]$

$= \dfrac{d}{dx}\left[\frac{1}{2}\log_5 (x^2-1)\right]$

$= \dfrac{1}{\cancel{2}} \dfrac{1}{(x^2-1)\ln 5}(\cancel{2}x) = \boxed{\dfrac{x}{(x^2-1)\ln 5}}$

55) $\dfrac{d}{dt}\left[\dfrac{10\log_4 t}{t}\right] = \dfrac{\cancel{t}\left(10 \cdot \dfrac{1}{\cancel{t}\ln 4}\right) - 10\log_4 t}{t^2} = \dfrac{\dfrac{10}{\ln 4} - \dfrac{10\ln t}{\ln 4}}{t^2} = \boxed{\dfrac{10(1-\ln t)}{t^2 \ln 4}}$