

1.8984375 E - 20

1. Shown above is a number displayed in scientific notation on a calculator. What is the 20th digit to the right of the decimal point when the number is expressed in decimal notation?

- (A) 0
- (B) 1
- (C) 5
- (D) 8

2. Juanita is 4 feet tall. Which of the following could be used to calculate her height in centimeters?
(1 inch = 2.54 centimeters)

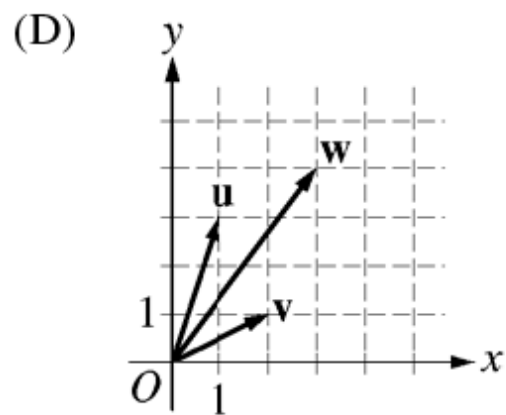
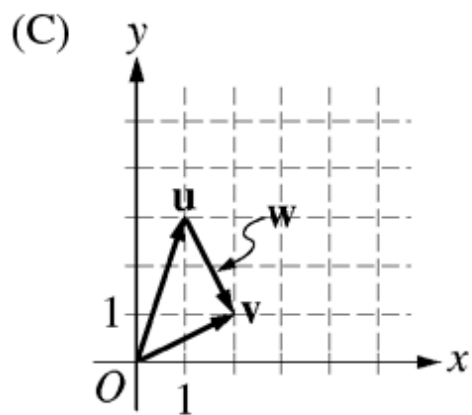
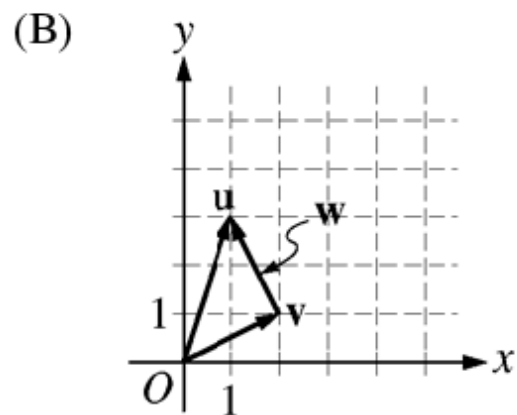
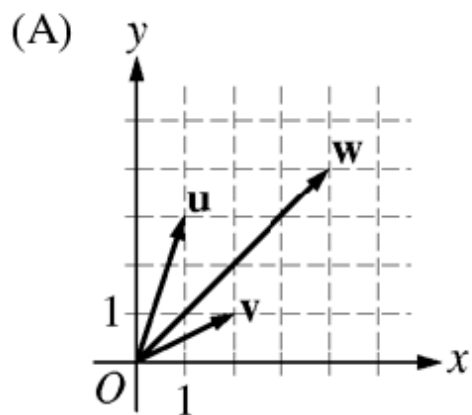
(A) $4 \text{ ft} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ in}}{2.54 \text{ cm}}$

(B) $4 \text{ ft} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{2.54 \text{ cm}}{1 \text{ in}}$

(C) $4 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{1 \text{ in}}{2.54 \text{ cm}}$

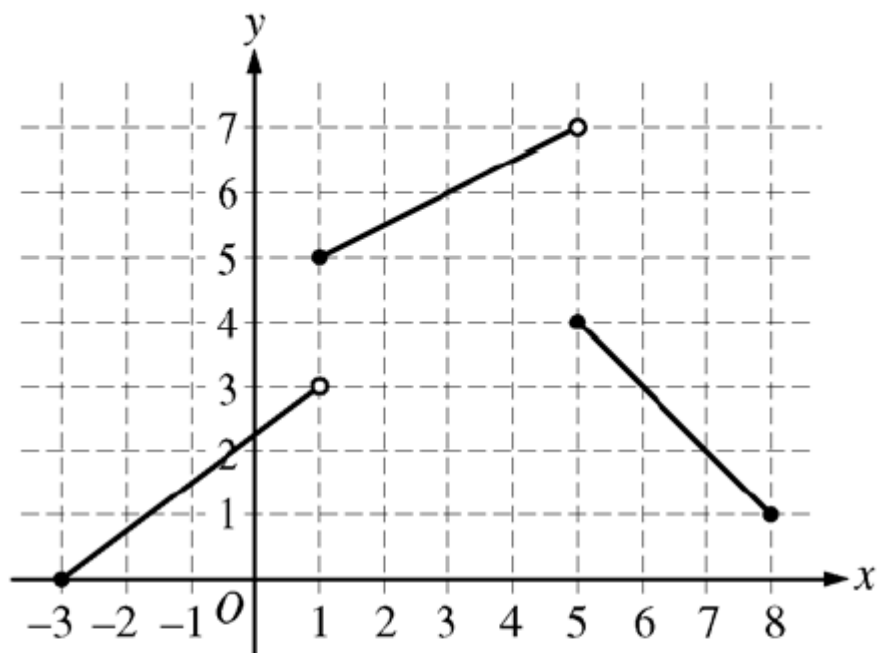
(D) $4 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}}$

3. Which of the following figures shows vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} in the xy -plane such that $\mathbf{w} = \mathbf{u} + \mathbf{v}$?



Contribution to Course Grade	Percent
Average (arithmetic mean) of chapter test scores	50%
Project score	20%
Final exam score	30%

4. In determining course grades, Ms. Rose calculates a weighted average as shown in the table above. If Frank's chapter test scores are 82, 75, and 79, his project score is 90, and his final exam score is 85, what is Frank's course grade, to the nearest tenth?
- (A) 82.2
(B) 82.8
(C) 83.1
(D) 83.7



5. The graph of the piecewise-linear function f is shown in the xy -plane above. Each of the following numbers is in the range of f EXCEPT
- (A) 3
 - (B) 3.5
 - (C) 4
 - (D) 4.5

6. In the xy -plane, the parabola with equation

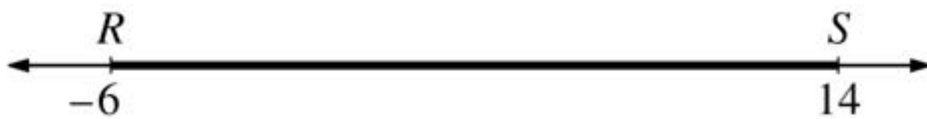
$y = x^2 - 4$ intersects the ellipse with equation

$\frac{x^2}{16} + \frac{y^2}{4} = 1$ in how many points?

- (A) None
- (B) One
- (C) Two
- (D) Four

7. In the xy -plane, which of the following points lies inside the circular region of radius 3 centered at $(1, -2)$?

- (A) $(-1, 1)$
- (B) $(1, 2)$
- (C) $(0, -5)$
- (D) $(3, 0)$



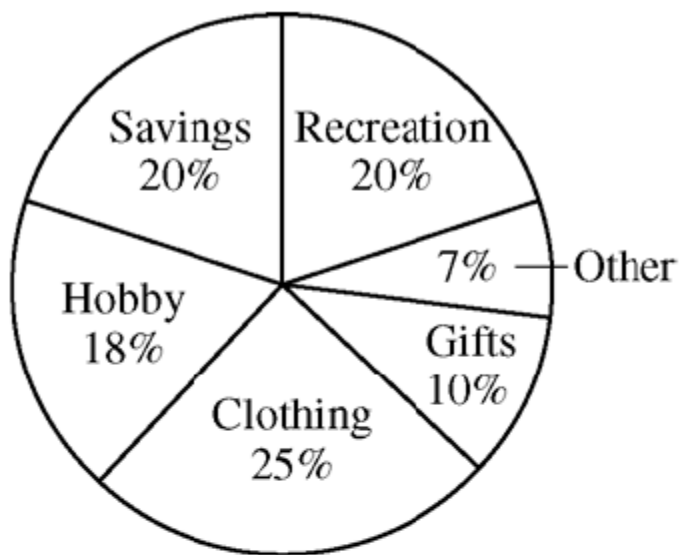
8. On the number line above, a point between points R and S will be selected at random. What is the probability that the point selected will be within 2 units of the point with coordinate 8 ?

(A) $\frac{1}{10}$

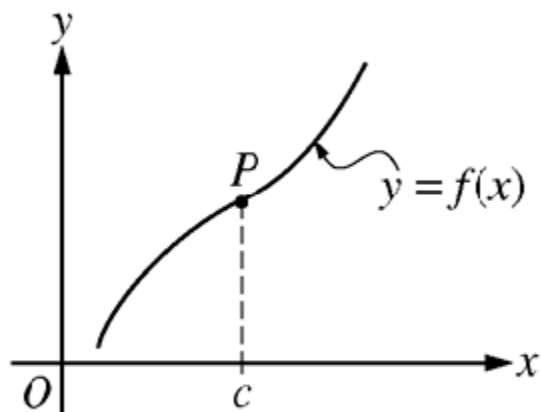
(B) $\frac{1}{8}$

(C) $\frac{1}{5}$

(D) $\frac{1}{4}$



9. The circle graph above shows how Maria spent her allowance in one year. She spent \$200 on recreation. Of the amount she spent on clothing, \$80 was spent on sweaters. What percent of the amount that she spent on clothing was the amount that she spent on sweaters?
- (A) 8%
(B) 25%
(C) 32%
(D) 40%



10. In the xy -plane above, point $P(c, f(c))$ is a point of inflection for the graph of the polynomial function f . In which of the following pairs of statements are both statements true?

- (A) $f''(c) = 0$ and $f'(c) > 0$
- (B) $f''(c) = 0$ and $f'(c) < 0$
- (C) $f''(c) > 0$ and $f'(c) = 0$
- (D) $f''(c) < 0$ and $f'(c) = 0$

11. The matrix representation for the linear transformation

$$\mathbf{T} : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ is } \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \text{ If } \mathbf{v} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \text{ then } \mathbf{T}(\mathbf{v}) =$$

(A) $\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$

(B) $\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$

(C) $\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

(D) $\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$

12. On a graphing calculator's viewing screen, the graph of $y = 4 \sin 2x \cos(-2x)$ would coincide with the graph of which of the following?

- (A) $y = 2 \sin 2x$
- (B) $y = 2 \sin 4x$
- (C) $y = 4 \sin 4x$
- (D) $y = -2 \cos 4x$

13. Event B happened after event A and before event C . The elapsed time between event A and event C was 10^9 seconds, and the elapsed time between event B and event C was 10^3 seconds. Which of the following is closest to the elapsed time, in seconds, between event A and event B ?

- (A) 10^9
- (B) 10^8
- (C) 10^6
- (D) 10^3

14. In the xy -plane, line ℓ has equation $y = -x$.
Point P lies on ℓ and has coordinates $(-2, 2)$.
If ℓ is rotated counterclockwise 45° about the origin, what will be the coordinates of the image of P under this rotation?
- (A) $(-2\sqrt{2}, 0)$
(B) $(-2, 0)$
(C) $(0, 2)$
(D) $(0, 2\sqrt{2})$
15. A new drug was tested on a group of 200 patients with high blood pressure. By the end of the trial period, 90 patients showed improvement from the drug, 35 patients developed side effects, and 95 patients showed no improvement and had no side effects. How many patients showed improvement and had side effects?
- (A) 15
(B) 20
(C) 25
(D) 30

	Jar 1	Jar 2
Red	4	5
Black	6	5

16. The table above shows the distribution of red and black chips in two jars. One chip is to be selected at random from each jar. What is the probability that both chips will be of the same color?

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{3}{5}$

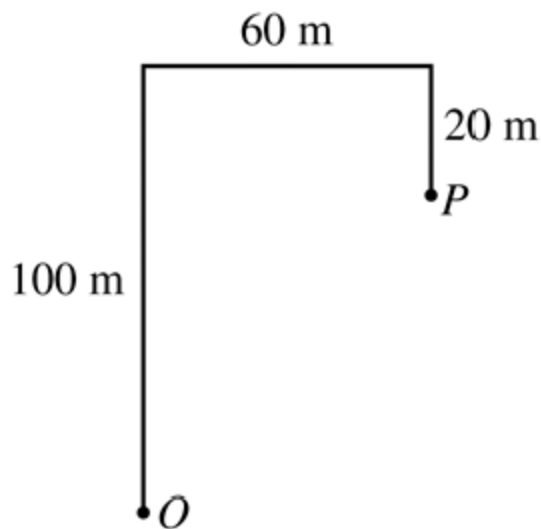
17. Changing the value of which of the positive constants a , b , c , and d will change the period of the function $y = a \sin(bx + c) + d$?

(A) a

(B) b

(C) c

(D) d



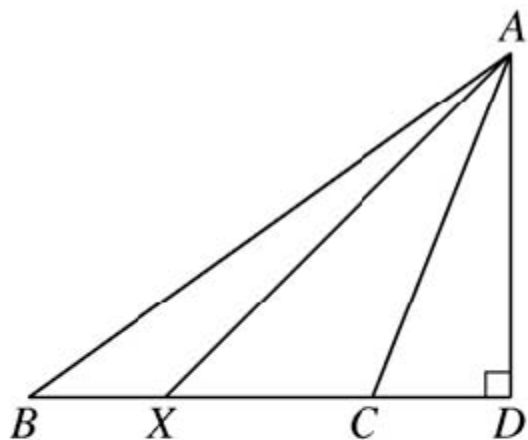
Note: Figure not drawn to scale.

18. In a lawn game, Jill walked from point O to point P , as shown in the figure above. She walked north for 100 meters, then east for 60 meters, and finally south for 20 meters. What is the distance between Jill's starting point and her ending point?
- (A) 80 m
(B) 90 m
(C) 100 m
(D) 120 m

19. Definition: The normal line to the curve $y = f(x)$ at the point P is the line perpendicular to the tangent line to the curve at P .

If an equation of the normal line to the graph of the function $y = f(x)$ at the point $(-3, 13)$ is $y = -2x + 7$, then $f'(-3)$ equals

- (A) -2
- (B) $-\frac{1}{2}$
- (C) $\frac{1}{2}$
- (D) 13
20. Which of the following complex numbers is the multiplicative identity for any complex number (a, b) ?
- (A) $(0, 0)$
- (B) $(0, 1)$
- (C) $(1, 0)$
- (D) $(1, 1)$



21. In triangle ABD above, angle D is a right angle and triangular regions ABC and ACD have area 180 and 90, respectively. If $BC = 20 = DX$, what is the length of XC ?

- (A) $6\frac{2}{3}$
(B) 10
(C) $13\frac{1}{3}$
(D) 15

22. If a , b , and c are real numbers such that

$\sqrt{b^2 - 4ac} = 0$, which of the following statements about the roots of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, must be true?

- (A) The equation has two distinct real roots.
- (B) The equation has two distinct nonreal roots.
- (C) The equation has one real root.
- (D) The equation has one nonreal root.

23. For a normally distributed population, approximately 68 percent of the population lies within 1 standard deviation of the mean and approximately 95 percent of the population lies within 2 standard deviations of the mean. In a certain environmental study, the weights of 10,000 fish are normally distributed with a mean of 12.5 ounces and a standard deviation of 4.2 ounces. Approximately what percent of the 10,000 fish have weights between 16.7 and 20.9 ounces?

- (A) 7%
- (B) 14%
- (C) 27%
- (D) 34%

$$a_{11}x + a_{12}y + a_{13}z = b_1$$

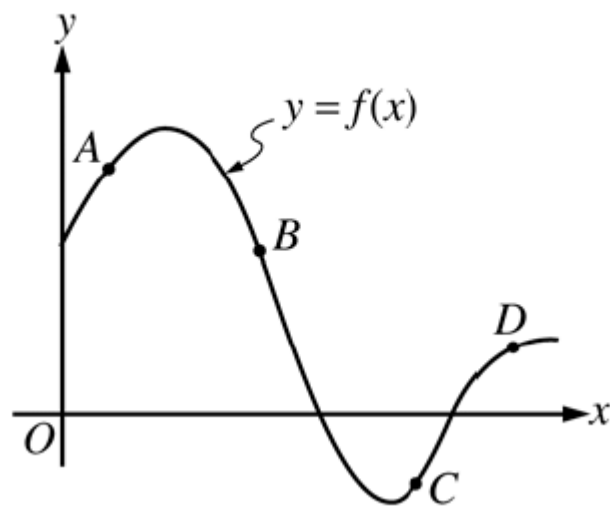
$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3$$

24. Let A be the matrix of coefficients a_{ij} of the system of equations above, and let B be the column vector whose entries are b_1 , b_2 , and b_3 . If the system of equations has a unique solution, then the solution is given by which of the following matrix products?

(Note: A^t denotes the transpose of A , and A^{-1} denotes the inverse of A .)

- (A) BA^{-1}
- (B) B^tA^{-1}
- (C) A^tB
- (D) $A^{-1}B$

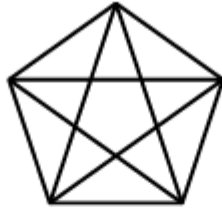


25. The graph of the continuous function f is shown in the xy -plane above. The function F (graph not shown) is defined by

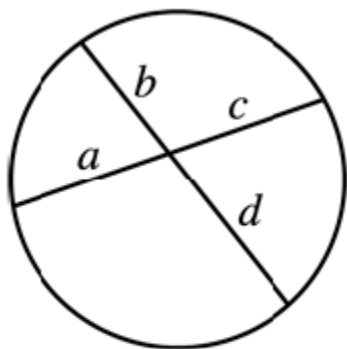
$$F(t) = \int_0^x f(t) dt.$$

At which of the labeled points is F decreasing?

- (A) A
- (B) B
- (C) C
- (D) D



26. In determining which of the following quantities is it LEAST appropriate to use the figure above as a model?
- (A) The number of games scheduled for 5 teams when each team is scheduled to play each of the other teams exactly once
 - (B) The number of different ways that 5 people can sit at a table having 5 sides, if one person sits at each side
 - (C) The number of 2-element subsets of a set of 5 elements
 - (D) The number of diagonals in a regular pentagon



27. In the figure above, the segments of each chord of the circle have the lengths shown. Which of the following must be true?

(A) $\frac{a}{c} = \frac{b}{d}$

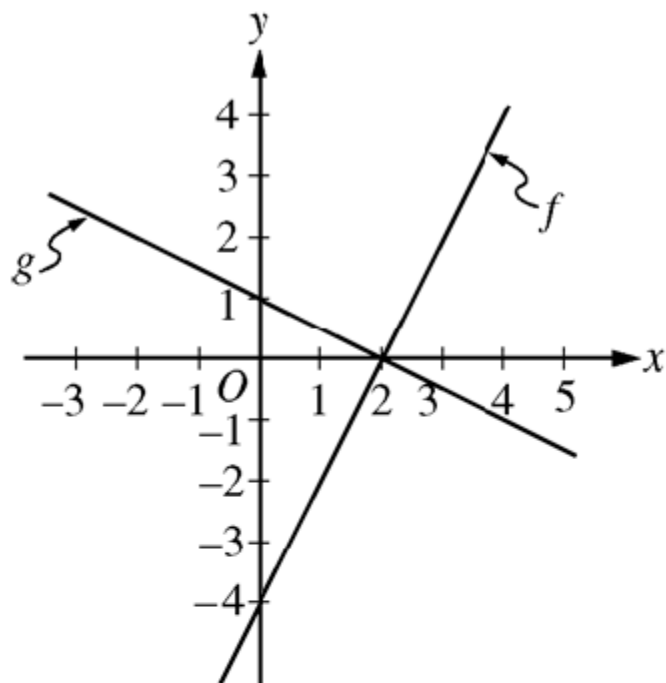
(B) $\frac{a}{d} = \frac{b}{c}$

(C) $\frac{a+c}{c} = \frac{b+d}{d}$

(D) $\frac{a+b}{c} = \frac{c+d}{b}$

28. In order to be included on the ballot in a certain city, a candidate must get at least 12.5% of the registered voters in each of three or more of the city's five districts to sign a petition. The ratio of the numbers of registered voters in the five districts is 6:5:5:4:5. In order for a candidate to be included on a ballot, what is the minimum percent of the total number of registered voters that must sign a petition?

- (A) 2%
- (B) 5%
- (C) 7%
- (D) 10%



29. The graphs of the linear functions f and g are shown in the xy -plane above. The linear function h is defined by $h(x) = f(g(x))$ for all x . What is the value of $h(2)$?
- (A) -4
(B) 0
(C) 1
(D) 2

30. The daily power requirement P , in megawatts, for the town of Kupinville can be modeled by the equation

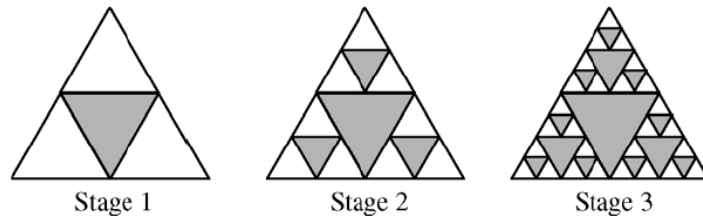
$$P(t) = 60 - 25 \cos\left(\frac{\pi}{12}t\right),$$

where t is the number of hours that have elapsed since midnight. At what time of the day is Kupinville's power requirement greatest?

- (A) Midnight
- (B) 6 A.M.
- (C) Noon
- (D) 6 P.M.

$$n = (2^{p-1})(2^p - 1)$$

31. The integer n has the form shown above, where both p and $2^p - 1$ are prime numbers and $p > 2$. What is the total number of positive divisors of n , including 1 and n , in terms of p ?
- (A) p
 - (B) $2p$
 - (C) p^2
 - (D) 2^{p-2}



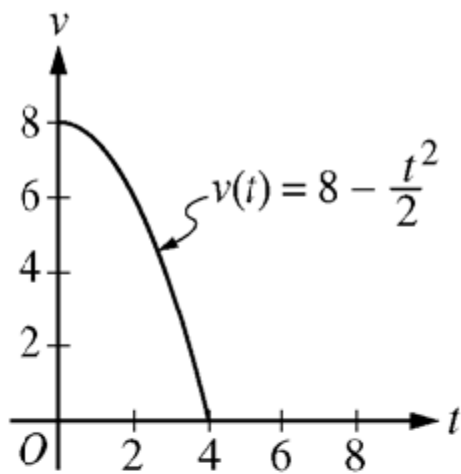
32. The figures above show the first three stages in the construction of a fractal. The construction starts with an unshaded equilateral triangle (not shown). At stage 1, the triangle with vertices at the midpoints of this triangle is shaded. At each subsequent stage, the process is repeated for each unshaded triangle in the previous stage. Which of the following statements is true about the perimeter P_n and area A_n of the shaded portion of the figure at stage n as $n \rightarrow \infty$?

- (A) Both P_n and A_n approach a finite limit.
- (B) Only P_n approaches a finite limit.
- (C) Only A_n approaches a finite limit.
- (D) Neither P_n nor A_n approaches a finite limit.

33. The operation \otimes is defined on the set \mathbb{Z}^+ of positive integers by $x \otimes y = x^y$ for all x and y in \mathbb{Z}^+ . Which of the following statements is true about the number 1 with respect to the operation \otimes ?

- (A) 1 is a right identity but not a left identity.
- (B) 1 is a left identity but not a right identity.
- (C) 1 is both a right and a left identity.
- (D) 1 is neither a right nor a left identity.

34. Three riders enter a train car that has 5 empty seats. Each rider will choose a different empty seat. How many possible arrangements are there of the 3 riders among the 5 seats?
- (A) 10
(B) 15
(C) 30
(D) 60
35. Let $f(x) = \sin x$, where x is a real number. The graphs of which of the following functions in the xy -plane intersect the graph of f in one or more points?
- I. $g(x) = 2 + \cos x$, where x is a real number
II. $h(x) = \ln(1 + x)$, where $x > -1$
- (A) Neither I nor II
(B) I only
(C) II only
(D) Both I and II



36. The velocity of an object, $v(t)$, in feet per second, is given by the equation

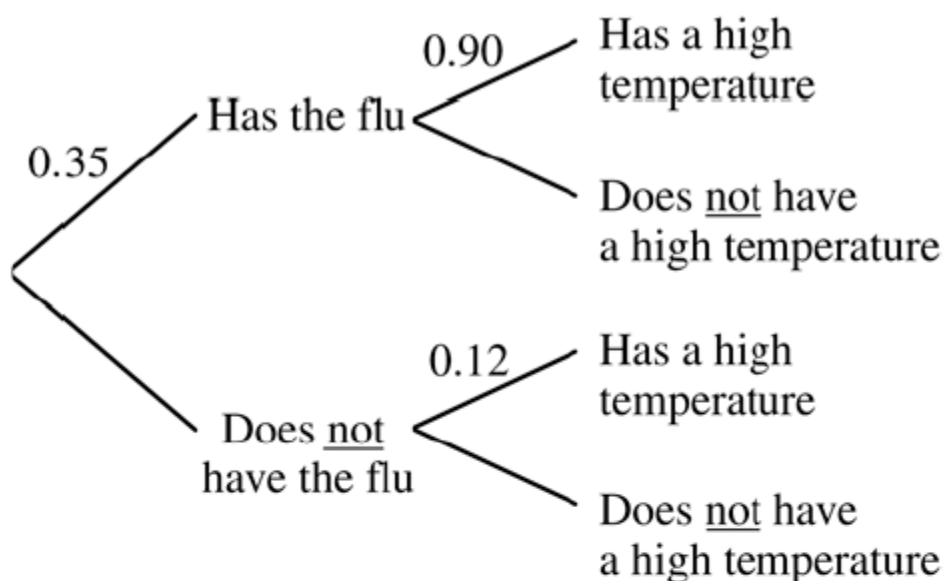
$$v(t) = 8 - \frac{t^2}{2},$$

where t is time, in seconds. The graph of v is shown in the coordinate system above. To the nearest foot, how far does the object travel in the first 2 seconds?

- (A) 6 ft
- (B) 7 ft
- (C) 13 ft
- (D) 15 ft

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

37. The linear transformation of the plane defined by the matrix equation above is a dilation of the plane. Under this transformation, the line $y = 3x + 2$ is mapped onto a line with slope
- (A) 3
 - (B) 4
 - (C) 8
 - (D) 12



38. The probability diagram above represents the incidence of high temperature among the students at a school who have and do not have the flu. For example, 0.90 is the probability that a student who has the flu also has a high temperature. What is the probability that a student has neither the flu nor a high temperature?
- (A) 0.470
(B) 0.572
(C) 0.685
(D) 0.770

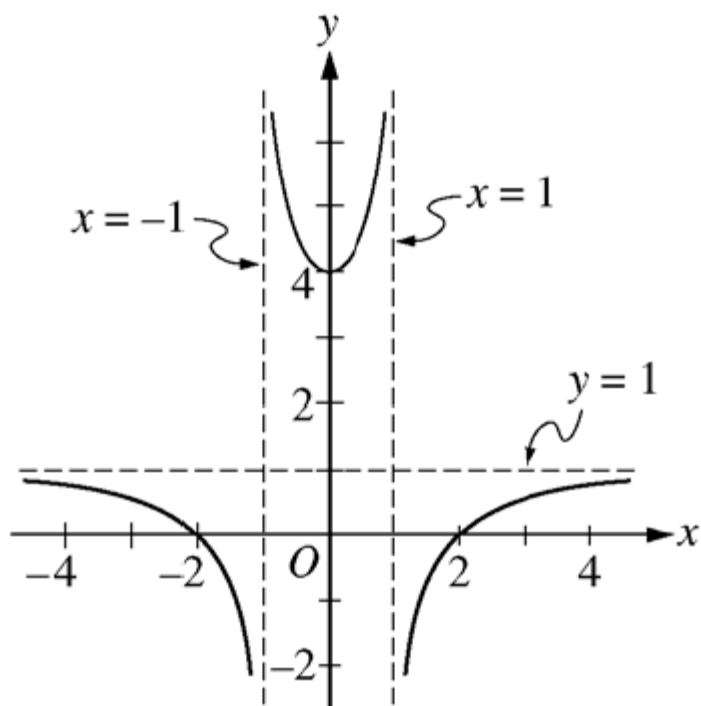
39. For a given scatterplot of data points (c, k) , a student transformed the data using a log scale on the vertical axis. The student then determined that a good fit for the transformed data points $(c, \ln k)$ was the regression line $y = 5.53 - 0.11c$, where $y = \ln k$. Based on the student's regression line, which of the following equations approximates the relationship between c and k ?

(A) $k = 252e^{-0.11c}$

(B) $k = 5.53e^{-0.11c}$

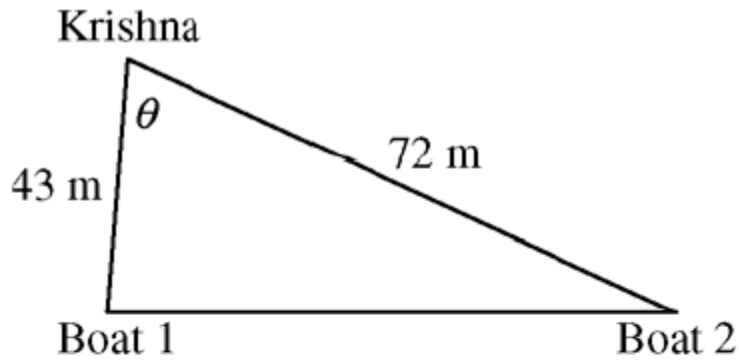
(C) $k = 5.53(1.11)^c$

(D) $k = e^{-\left(\frac{c-5.53}{0.11}\right)^2}$



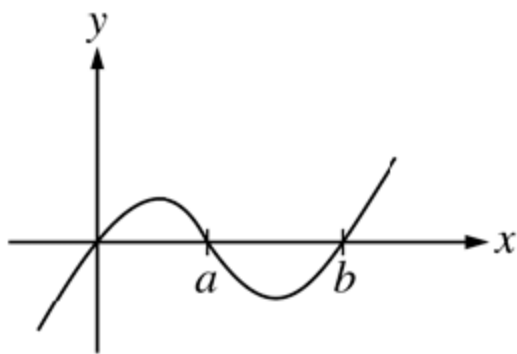
40. The xy -plane above shows the three branches of the graph of $y = f(x) = \frac{ax^2 + b}{x^2 + c}$, where a , b , and c are constants. If $\lim_{x \rightarrow 1^-} f(x) = +\infty$, what is the value of c ?

- (A) -1
- (B) 1
- (C) 2
- (D) 4



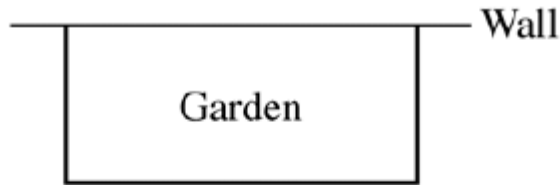
Note: Figure not drawn to scale.

41. Standing on a bridge, Krishna sees two boats, as shown above. The angle, θ , between his line of sight to each boat is 58° , and the boats are 43 meters and 72 meters away, respectively, along his line of sight. What is the distance between the two boats?
- (A) 41.69 m
(B) 61.25 m
(C) 83.86 m
(D) 101.56 m



42. The graph of the function $y = f(x)$ is shown in the xy -plane above. Which of the following could be an equation for $y = f(x)$?

- (A) $y = (x - a)(x - b)$
- (B) $y = x(x - a)(x - b)$
- (C) $y = x(x - a)(x + b)$
- (D) $y = x(x + a)(x + b)$



Note: Figure not drawn to scale.

43. Claire wants to fence in a rectangular plot of land along a wall to make a garden. She has p feet of fencing, which will extend along three sides of the garden, with the wall forming the fourth side, as shown in the figure above. Which of the following expressions in p is equal to the maximum possible area, in square feet, of Claire's garden?

(A) $2p^2$

(B) $\frac{p^2}{2}$

(C) $\frac{p^2}{4}$

(D) $\frac{p^2}{8}$

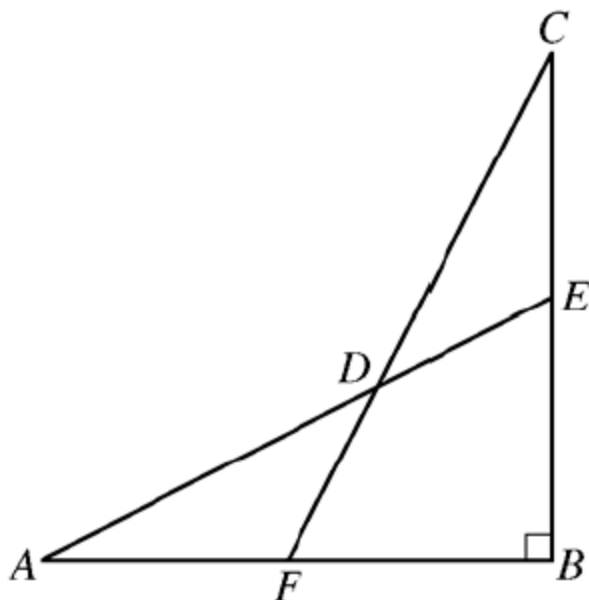
44. The graph of the function $f(x) = x^2$ in the xy -plane is translated three units to the right and one unit up to create a graph of the function g . At what point do the graphs of f and g intersect?

(A) $\left(\frac{5}{3}, \frac{25}{9}\right)$

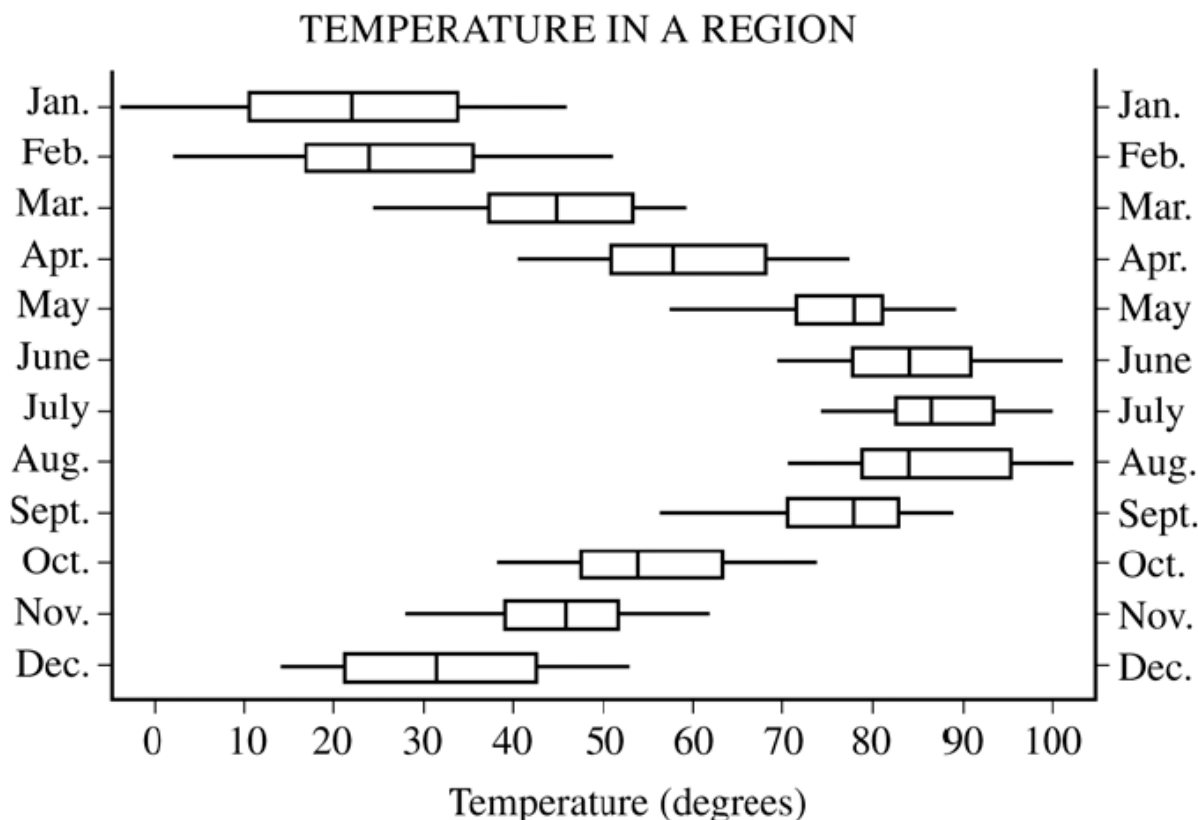
(B) $\left(-\frac{5}{3}, \frac{25}{5}\right)$

(C) $\left(\frac{4}{3}, \frac{16}{9}\right)$

(D) $\left(-\frac{4}{3}, \frac{16}{9}\right)$



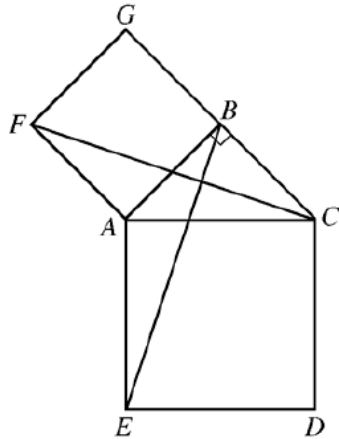
45. In the figure above, $AB = BC = 2$, point E is the midpoint of \overline{BC} , and point F is the midpoint of \overline{AB} . What is the measure, to the nearest degree, of $\angle FDE$?
- (A) 130°
(B) 143°
(C) 150°
(D) 153°



46. The graph above shows boxplots of daily high temperatures in a certain region for each of twelve consecutive months. Based on these boxplots, which of the following statements about the temperature in the region during the twelve-month period is true?
- (A) It was colder in the region in June than in September.
 - (B) The range of the daily high temperatures for August was greater than the corresponding range for January.
 - (C) The daily high temperature in February never exceeded that in April.
 - (D) The median daily high temperature for the months of April through October was above 50 degrees.

47. The graph of $y = xe^x$ on the xy -plane has

- (A) a vertical asymptote
- (B) positive slope for all real x
- (C) a relative maximum
- (D) an absolute minimum



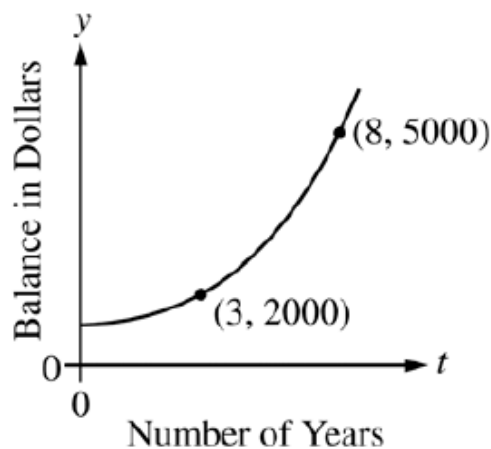
Given: (1) ABC is a right triangle.
 (2) $AFGB$ and $ACDE$ are squares.

Prove: $\overline{FC} \cong \overline{BE}$

Statement	Reason
1. $\overline{FA} \cong \overline{BA}$	All sides of a square are congruent.
2. $\overline{AC} \cong \overline{AE}$	All sides of a square are congruent.
3. $\angle FAB \cong \angle CAE$	All right angles are congruent.
4. $\angle BAC \cong \angle BAC$	Reflexive property of congruence
5. $\angle FAC \cong \angle BAE$	Angle addition postulate
6. $\triangle FAC \cong \triangle BAE$?
7. $\overline{FC} \cong \overline{BE}$	Corresponding parts of congruent triangles are congruent.

48. The figure above consists of a right triangle and squares on two of the sides. The table gives all but one of the reasons for the statements used to prove that $\overline{FC} \cong \overline{BE}$. Which of the following is the missing reason for statement 6 ?

- (A) SSS
- (B) SAS
- (C) SAA
- (D) ASA



Note: Figure not drawn to scale.

49. An investor deposited a certain amount of money in a new account at an annual interest rate of r , compounded continuously. The figure above shows the graph of $y = p(t)$, where $p(t)$ is the balance, in dollars, of the account t years after the investor made the deposit and $t \geq 0$. If no other transactions were made in the account, which of the following is closest to the amount, in dollars, the investor deposited in the account at time $t = 0$? (Note: $p(t) = p_0 e^{rt}$)
- (A) \$540
 (B) \$1,000
 (C) \$1,160
 (D) \$1,270

$$b_1, b_2, b_3, b_4, \dots$$

50. In the geometric sequence above, $b_1 = 1,000$ and

$$b_n = \frac{2}{3}b_{n-1} \text{ for all } n \geq 2. \text{ What is the least value}$$

of k for which $b_k < 0.001$?

- (A) 30
- (B) 32
- (C) 34
- (D) 36

Sequence Number	Correct Answer	Content Category
1	B	I
2	D	II
3	D	V
4	B	IV
5	D	III
6	D	I
7	D	I
8	C	IV
9	C	IV
10	A	III
11	A	V
12	B	II
13	A	II
14	A	II
15	B	V
16	C	IV
17	B	II
18	C	II
19	C	III
20	C	I
21	B	II
22	C	I
23	B	IV
24	D	V
25	C	III

Sequence Number	Correct Answer	Content Category
26	B	V
27	B	II
28	C	I
29	A	III
30	C	III
31	B	I
32	C	II
33	A	I
34	D	V
35	C	III
36	D	III
37	A	V
38	B	IV
39	A	IV
40	A	III
41	B	II
42	B	III
43	D	III
44	A	III
45	B	II
46	D	IV
47	D	III
48	B	II
49	C	III
50	D	V