

# Rectangular Approximation and the Trapezoidal Rule

1. (Calculator)

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$t$ (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function  $R$  of time  $t$ . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate  $\int_0^{24} R(t)dt$ . Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time  $t$ ,  $0 < t < 24$ , such that  $R'(t) = 0$ ? Justify your answer.
- (c) The rate of water flow  $R(t)$  can be approximated by  $Q(t) = \frac{1}{79}(768 + 23t - t^2)$ . Use  $Q(t)$  to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

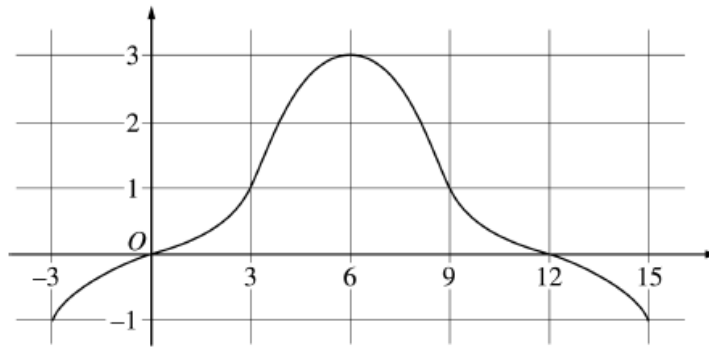
2. (Calculator)

$t$ (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function  $W$  of time  $t$ . The table above shows the water temperature as recorded every 3 days over a 15-day period.

- Use data from the table to find an approximation for  $W'(12)$ . Show the computations that lead to your answer. Indicate units of measure.
- Approximate the average temperature, in degrees Celsius, of the water over the time interval  $0 \leq t \leq 15$  days by using a trapezoidal approximation with subintervals of length  $\Delta t = 3$  days.
- A student proposes the function  $P$ , given by  $P(t) = 20 + 10te^{(-t/3)}$ , as a model for the temperature of the water in the pond at time  $t$ , where  $t$  is measured in days and  $P(t)$  is measured in degrees Celsius. Find  $P'(12)$ . Using appropriate units, explain the meaning of your answer in terms of water temperature.
- Use the function  $P$  defined in part (c) to find the average value, in degrees Celsius, of  $P(t)$  over the time interval  $0 \leq t \leq 15$  days.

3. (Non-Calculator)



Graph of  $f$

The graph of a differentiable function  $f$  on the closed interval  $[-3, 15]$  is shown in the figure above. The graph of  $f$  has a horizontal tangent line at  $x = 6$ . Let  $g(x) = 5 + \int_6^x f(t) dt$  for  $-3 \leq x \leq 15$ .

- Find  $g(6)$ ,  $g'(6)$ , and  $g''(6)$ .
- On what intervals is  $g$  decreasing? Justify your answer.
- On what intervals is the graph of  $g$  concave down? Justify your answer.
- Find a trapezoidal approximation of  $\int_{-3}^{15} f(t) dt$  using six subintervals of length  $\Delta t = 3$ .

## 4. (Calculator)

$t$ (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per minute at time  $t$  minutes, where  $v$  is a differentiable function of  $t$ . Selected values of  $v(t)$  for  $0 \leq t \leq 40$  are shown in the table above.

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to

approximate  $\int_0^{40} v(t) dt$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $\int_0^{40} v(t) dt$  in terms of the plane's flight.

- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval  $0 < t < 40$ ? Justify your answer.

- (c) The function  $f$ , defined by  $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$ , is used to model the velocity of the plane, in miles per minute, for  $0 \leq t \leq 40$ . According to this model, what is the acceleration of the plane at  $t = 23$ ? Indicate units of measure.

- (d) According to the model  $f$ , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 \leq t \leq 40$ ?

5. (Calculator)

Distance $x$ (cm)	0	1	5	6	8
Temperature $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature  $T(x)$ , in degrees Celsius ( $^{\circ}\text{C}$ ), of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.

- (a) Estimate  $T'(7)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of  $T(x)$  for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find  $\int_0^8 T'(x) dx$ , and indicate units of measure. Explain the meaning of  $\int_0^8 T'(x) dx$  in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that  $T''(x) > 0$  for every  $x$  in the interval  $0 < x < 8$ ? Explain your answer.