AP Calculus Transcendental Practice

1969 AB

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \dfrac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

20. An equation for a tangent to the graph of $y = \arcsin\dfrac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

22. $\dfrac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\dfrac{1}{e^{2x}}$ (B) $\dfrac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

38. $\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

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2. What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?

(A) $(-1,0)$ (B) $(0,0)$ (C) $(0,1)$ (D) $\left(1,\frac{\pi}{4}\right)$ (E) $\left(1,\frac{\pi}{2}\right)$

10. $\int_0^1 \frac{x^2}{x^2+1}\,dx =$

(A) $\frac{4-\pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\frac{1}{2}\ln 2$ (E) $\frac{4+\pi}{4}$

12. If $F(x)=\int_0^x e^{-t^2}\,dt$, then $F'(x)=$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1}-e$

(D) $e^{-x^2}-1$ (E) e^{-x^2}

23. If the graph of $y=f(x)$ contains the point $(0,2)$, $\dfrac{dy}{dx}=\dfrac{-x}{ye^{x^2}}$ and $f(x)>0$ for all x, then $f(x)=$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

28. What is $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

38. If $f(x)=\left(x^2+1\right)^{(2-3x)}$, then $f'(1)=$

(A) $-\dfrac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\dfrac{3}{2}\ln(2)$ (D) $-\dfrac{1}{2}$ (E) $\dfrac{1}{8}$

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3. The slope of the line tangent to the graph of $y=\ln\left(x^2\right)$ at $x=e^2$ is

(A) $\dfrac{1}{e^2}$ (B) $\dfrac{2}{e^2}$ (C) $\dfrac{4}{e^2}$ (D) $\dfrac{1}{e^4}$ (E) $\dfrac{4}{e^4}$

15. The area of the region bounded by the lines $x=0$, $x=2$, and $y=0$ and the curve $y=e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t=0$, the number of bacteria present was 7,500. Find the number present at $t=5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

18. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

30. $\int_1^2 \frac{x-4}{x^2}dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

31. If $\log_a\left(2^a\right)=\frac{a}{4}$, then $a=$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

37. If $\frac{dy}{dx}=4y$ and if $y=4$ when $x=0$, then $y=$

(A) $4e^{4x}$ (B) e^{4x} (C) $3+e^{4x}$ (D) $4+e^{4x}$ (E) $2x^2+4$

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1. If $f(x)=e^{1/x}$, then $f'(x)=$

(A) $-\frac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\frac{e^{1/x}}{x}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x}e^{(1/x)-1}$

7. If $y=\ln\left(x^2+y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

31. If $f(x)=\ln(\ln x)$, then $f'(x)=$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x\ln x}$

32. If $y = x^{\ln x}$, then y' is

(A) $\dfrac{x^{\ln x} \ln x}{x^2}$

(B) $x^{1/x} \ln x$

(C) $\dfrac{2x^{\ln x} \ln x}{x}$

(D) $\dfrac{x^{\ln x} \ln x}{x}$

(E) None of the above

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7. Which of the following is equal to $\ln 4$?

(A) $\ln 3 + \ln 1$ (B) $\dfrac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t \, dt$ (D) $\int_1^4 \ln x \, dx$ (E) $\int_1^4 \dfrac{1}{t} dt$

8. The slope of the line tangent to the graph of $y = \ln\left(\dfrac{x}{2}\right)$ at $x = 4$ is

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 1 (E) 4

9. If $\int_{-1}^{1} e^{-x^2} dx = k$, then $\int_{-1}^{0} e^{-x^2} dx =$

(A) $-2k$ (B) $-k$ (C) $-\dfrac{k}{2}$ (D) $\dfrac{k}{2}$ (E) $2k$

10. If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{(x^2-1)}$ (B) $(2x)10^{(x^2-1)}$ (C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$ (E) $x^2(\ln 10)10^{(x^2-1)}$

20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0} \frac{e^{x+h}-e^e}{h}$ (C) $\lim_{h\to 0} \frac{e^{e+h}-e}{h}$

(D) $\lim_{h\to 0} \frac{e^{x+h}-1}{h}$ (E) $\lim_{h\to 0} \frac{e^{e+h}-e^e}{h}$

30. $\int \tan(2x)\,dx =$

(A) $-2\ln|\cos(2x)|+C$ (B) $-\frac{1}{2}\ln|\cos(2x)|+C$ (C) $\frac{1}{2}\ln|\cos(2x)|+C$

(D) $2\ln|\cos(2x)|+C$ (E) $\frac{1}{2}\sec(2x)\tan(2x)+C$

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7. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}\,dx$?

(A) $\arcsin\frac{x}{5}+C$ (B) $\arcsin x+C$ (C) $\frac{1}{5}\arcsin\frac{x}{5}+C$

(D) $\sqrt{25-x^2}+C$ (E) $2\sqrt{25-x^2}+C$

11. $\frac{d}{dx}\ln\left(\frac{1}{1-x}\right)=$

(A) $\frac{1}{1-x}$ (B) $\frac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

17. If $f(x)=x\ln\left(x^2\right)$, then $f'(x)=$

(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$ (D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

28. An antiderivative of $f(x)=e^{x+e^x}$ is

(A) $\frac{e^{x+e^x}}{1+e^x}$ (B) $\left(1+e^x\right)e^{x+e^x}$ (C) e^{1+e^x} (D) e^{x+e^x} (E) e^{e^x}

33. If $\frac{dy}{dt}=-2y$ and if $y=1$ when $t=0$, what is the value of t for which $y=\frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$